Fed Funds Futures And The Federal Reserve

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Abstract

This paper introduces a novel affine dynamic term structure model estimated using daily futures, LIBOR and target rates. The results uncover substantial cyclical changes in the uncertainty surrounding target changes that are induced by the Fed's response to economic conditions. The uncertainty is lowest (highest) in tightening (loosening) cycles, especially when the economy emerges from (enters) a recession. This adds to risk premium variations over the impact of changes in the price of macro risk. The model correctly characterizes risk premium since (i) it fits interest rates more accurately and (ii) it delivers unbiased target rate forecasts that match or improve upon standard benchmarks, including predictive regressions based on federal funds futures. The information content of Fed funds futures have been neglected in the term structure literature, perhaps due to technical challenges. The paper traces this predictive content to the ability hedging demands in the futures market.

Keywords: Affine Models, Federal Reserve, fed funds futures, LIBOR rates, risk premium, liquidity premium
JEL Classifications: E43, E44, E47, G12, G13

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I Introduction

Prices of futures contracts on the overnight federal funds rate are widely used to measure monetary policy expectations in the US. Indeed, predictive OLS regressions based on futures prices produce unbiased forecasts of target rates at very short horizons (Hamilton (2009)) and still produce the most accurate forecasts whenever a liquid contract is available (Gurkaynak et al. (2007)). An important implication is that futures rates offer the greatest potential to help characterize the Federal Reserve [Fed] policy function and pin down the Fed's policy rate dynamics. That potential has been neglected in the term structure literature. This paper contributes a novel dynamic term structure model that exploits the information content of futures rates and the discrete nature of target changes. It offers three significant empirical contributions.

First, it reveals the cyclical uncertainty surrounding the Fed's policy decisions. The volatility of target rate changes is lowest during or in anticipation of a tightening cycle. Conversely, the volatility is highest during or in anticipation of a loosening cycle. Also, volatility is negatively correlated with the level of the target rate but this effect is quantitatively smaller. Finally, target rate innovations are significantly skewed upward in a tightening cycle (where volatility is low) but skewed only slightly downward in a loosening cycle. In other words, the Fed is perceived to act more rapidly and with less uncertainty when leaning against inflation risk in a growing economy than when supporting employment in a slowing economy. Note that variations in the volatility (and skewness) of policy shocks arise from the novel response function introduced here. The relationship between state variables and target rate uncertainty is characterized by the same parameters that determine the conditional mean of target changes. Moreover, the effect arises even if volatility does not have its own shock and the underlying state variables have constant variance. This represents a novel channel through which policy affects long interest rates. In particular, the reduced variance of policy shocks in the early stages of a tightening cycle lowers the risk premium and mutes the transmission of target rate hikes to longer interest rates. Conversely, the increased variance in the early stages of a recession raises the risk premium and also mutes the effect of policy. This channel is separate from the direct effect of improved economic conditions on the risk premium. Importantly, it adds to the trade-offs faced by the Fed in its choice of policy rule. The implications for welfare have not been addressed so far in the literature.

Second, the model measures the ex-ante counter-cyclical risk premium implicit in interest rates. First, the model matches LIBOR and futures rates and, therefore, captures the risk-neutral dynamics implicit in the cross-section of rates. Second, the model delivers accurate and unbiased target rate forecasts and, therefore, captures the historical dynamics as well. In particular, the model matches benchmark predictive regressions based on future rates and improves upon regressions based on other market rates at horizons beyond 6 months (where futures quotes are not consistently available). Together, these results imply accurate estimates of the adjustment for risk between expectations under the historical measure and expectations under the risk-neutral measure : the risk premium. Risk premium variations are in part due to changes in the price of macro risk but also to the cyclical changes of uncertainty. They blur unadjusted market-based forecasts (Piazzesi and Swanson (2006)) leading to upward bias when we enter a loosening cycle and downward bias when we enter a tightening cycle. In other words, unadjusted forecast errors are larger in magnitude at both ends of business cycles, precisely when they matter most to investors and policy-makers. In contrast with predictive regressions, the no-arbitrage restrictions combined with the structure of the model allows us to disentangle counter-cyclical risk premium variations from the large excess returns realized from position in the futures market. These returns follow the policy response to recessionary shocks. Predictive OLS regressions cannot achieve this decomposition without very long sample, tend to over-fit these episodes with large returns, giving weight to events that were unpredicted.

Third, I ask why the combination of futures and LIBOR rates adds to our characterization of the risk premium and of the target rate dynamics. First, spreads between futures and forward LIBOR rates share a factor structure across maturities. Two-thirds of their total variations is summarized by a level and a slope principal component. Second, the slope component has substantial predictive power for excess returns from futures or forward positions and, therefore, can be used to improve target rate forecasts. Third, the model captures this information of forward-to-futures spreads via a latent factor, akin to a convenience yield. But why are some market rates more informative than others? Higher forward-to-futures spreads may predict target rates if the Fed responds to credit or funding conditions in the LIBOR market. Alternatively, consistent with Piazzesi and Swanson (2006), the hedging demand of commercial banks may cause transitory price pressures in futures rates and reveals expectations of target rate changes. Both channels imply a negative correlation between forward-to-futures spreads and target changes. On the other hand, the two channels offer opposite implications for the correlation between spreads and excess returns. Higher spreads due to higher compensation for default or funding risk are *positively* correlated with excess returns in LIBOR rates *only*. In contrast, higher spreads due to hedging pressures in the futures market are *negatively* correlated with excess returns in futures rates *only*. I test each implication via predictive regressions. Overall, the evidence of excess returns predictability is much stronger than previously documented and, more importantly, it unambiguously points at futures markets as the source of target rate predictability. Transitory deviations of futures rates, relative to a frictionless no-arbitrage model, contain substantial information about future target rates. Also, the information of forward-to-futures spreads overlap with the information content of net trading positions of non-commercial hedgers. Anticipating on the results, I label the latent factor capturing the information content of spreads a liquidity factor.[1]

Federal funds futures contracts have failed to attract attention from the term structure literature, perhaps because canonical affine models (Dai and Singleton (2000)) perform poorly at short maturities (see e.g. Piazzesi (2005a)). Then, on the one hand, we have that futures predict policy rates and, on the other hand, current models struggle to link futures with policy rate dynamics (via the no-arbitrage restriction). The source of this apparent contradiction lies in the Fed's operational procedure. Target rates exhibit a step-like path where changes take a small number of values and occur following a Federal Open Market Committee [FOMC] meeting. There is at most one scheduled meeting in a given month and pricing futures amounts to deciding on the probabilities of the few possible outcomes from that meeting. In contrast, standard term structure models attribute positive probabilities across a continuous range of outcomes for all days.

The empirical contributions in this paper are based on a novel discrete-time no-arbitrage term structure model that matches the properties of the Feds target rate. Conditional on state variables, the policy decision has a Skellam (Skellam (1946)) distribution over a discrete support. Intuitively, this distribution is constructed from the difference between two independent Poisson random variables. This approach is most closely related to Piazzesi (2005a), who models the target rate as the difference between two pure jump processes, but it offers several extensions and advantages. First, although futures rates are not affine, they can be computed explicitly with coefficients given by known recursions. The simplicity of the recursions contrasts with the continuous-time case where the coefficients would be solutions to non-standard differential equations. Second, I avoid the simplifying assumption that future FOMC meetings occur at a regular interval. This is relevant for pricing futures but it implies that the target rate is not Markov since the schedule is not time-homogenous. However, this can be handled relatively easily in discrete-time, compounding the advantages when pricing futures. Third, I allow for cyclical uncertainty around target changes. Fourth, I allow for a flexible affine specification of the prices of risk. In particular, the price of target rate risk is not restricted to zero and the risk premium may vary with uncertainty. Finally, the model is readily interpretable and has a Vector Auto-Regressive [VAR(1)] representation. In particular, it is amenable to implementations in a structural framework although I leave this avenue for future research.

Balduzzi et al. (1997) and Balduzzi et al. (1997) first investigated the implications of discrete target changes for the term structure of interest rates. Hamilton and Jordà (2002) combine an

[1] I use this label for ease of exposition, to contrast with an interpretation based on variations in the default component of LIBOR rates and to emphasize the evidence suggesting that this factor arises due to the limited ability of non-bank participants to absorb the hedging demand of banks.

ordered response function with a conditional hazard rate but do not impose no-arbitrage restrictions (see also Grammig and Kehrle (2008)). They conclude that interest rates are key conditioning variables for target rate forecasts but that forecasting at long horizons remains a challenge since rates are themselves difficult to predict. A no-arbitrage model uses interest rates at different maturities and forecasts consistently at any horizon. Krueger and Kuttner (1996) were first to consider target rate forecasts based on futures contracts. Since, Gurkaynak et al. (2007) have shown that forecasts based on fed funds futures and eurodollar futures outperform. Hamilton (2009) presents evidence that daily changes in near-maturity futures prices accurately reflect changes in the market's expectations of future policy rates. Piazzesi and Swanson (2006) argue that futures with longer maturities include a significant, counter-cyclical risk premium (see also Sack (2004)). On the other hand, Hamilton and Okimoto (2010) analyze a regime-switching model and argue that most of the variations in holding returns are confined to episodes that characterized by higher mean and volatility of excess returns. I offer a reconciliation between these explanations. Estimates of the risk premium do exhibit significant counter-cyclical variations throughout the sample. Nonetheless, the model attributes the largest and most volatile excess returns to errors in forecasting aggressive loosening cycles or sudden stop in a tightening cycle. These correspond to that regime with higher mean and volatility in Hamilton and Okimoto (2010).

Kuttner (2001) uses futures to measure unanticipated target changes and their impacts on interest rates at longer maturities. Similarly, Bernanke and Kuttner (2005) document the impact of unanticipated target changes on stock prices. Rudebusch (1998) show that policy shocks obtained from recursive identification of VAR residuals show little correlation with shocks measured from futures. Faust et al. (2003) show that structural VARs based on these two identification strategies can have different implications (see also Cochrane and Piazzesi (2002)). Moreover, recursive identification is hard to justify when including more than one financial variable. D'amico and Farka (2011) uses the schedule of FOMC meetings and high-frequency futures data around FOMC data to identify policy shocks. In all cases, the conclusions rely on the assumption that daily changes in the risk premium are unimportant. Then, only the shortest maturity can be used. The methodology introduced here opens the way for combining no-arbitrage restrictions with structural VAR assumptions to address the joint determination of the target rate, interest rates and economic variables.

The results contrast with those from existing specifications where volatility is either constant or, if stochastic, is uncorrelated with economic state variables. Term structure models based on square-root processes (e.g. Cox et al. (1985)), regime changes (e.g. Bansal and Zhou (2004)), quadratic processes (e.g. Ahn et al. (2002) or stochastic volatility processes allow for changes in volatility which are unrelated to the policy response function. In the context of structural VAR models, Cogley and Sargent (2005) allow for drifting volatility coefficient and Sims and Zha (2006) allow for regime changes but they do not address the implications for the term structure of interest rates. Recently, Ang et al. (2009) study the implications for the term structure of a response function with time-varying coefficients that may be correlated with economic conditions. On the other hand, they do not address changes in the volatility of policy shocks, the discrete nature of target rate changes, or the information content of futures. Finally, the federal funds market is covered by a large literature.[2] Hamilton (1996) highlights the importance of jumps and of time-varying volatility (see also Das (2002)) and finds that persistent jumps are associated with FOMC meetings. Johannes (2004) provides conclusive evidence that persistent jumps in fed fund rates affect the evolution of other short-term rates.

The rest of the paper is organized as follows. Section II summarizes the data and discusses the information content of forward-to-futures spreads. Section III introduces the model, discusses its VAR representation under each measure and derives asset pricing implicationss. Section IV provides a state-space representation and presents estimation results with an emphasis on target rate forecasts and the determinants of the risk premium. Details of the estimation methodology are relegated to the appendix. Section V traces the predictive content obtained from combining futures and LIBOR rates to hedging demand in the market for fed funds futures. Section VI concludes.

II Federal Funds, Futures and Libor Rates

A *Data*

I use a sample of daily target and effective rates on overnight funds at the Federal Reserve from the beginning of 1994 to the end of July 2007. I also use daily data on fed funds futures contracts with horizons from 1 to 6 months and LIBOR rates with maturities of 1 to 12 months. LIBOR rates provide a natural term structure associated with the overnight market since they correspond to the rates at which large banks are prepared to lend to each other on an unsecured basis. I exclude futures contracts with horizons beyond 6 months since they are relatively illiquid for most of the sample and there are many days with no transactions. I use end-of-the-day target and effective overnight rates from the Federal Reserve Bank of New York and futures rates from Datastream. LIBOR rates are published around 11h30 by the British Bankers Association in London. I match LIBOR data with fed funds and futures rates from the previous day. More details about the data is provided in Appendix A.1.

[2]See Hamilton (1996) and Stigum (2004) for detailed discussions of the fed funds market.

Figure 1: **Target Federal Funds Rate and 1-year constant maturity Treasury yields.** Daily data (1994-2007).

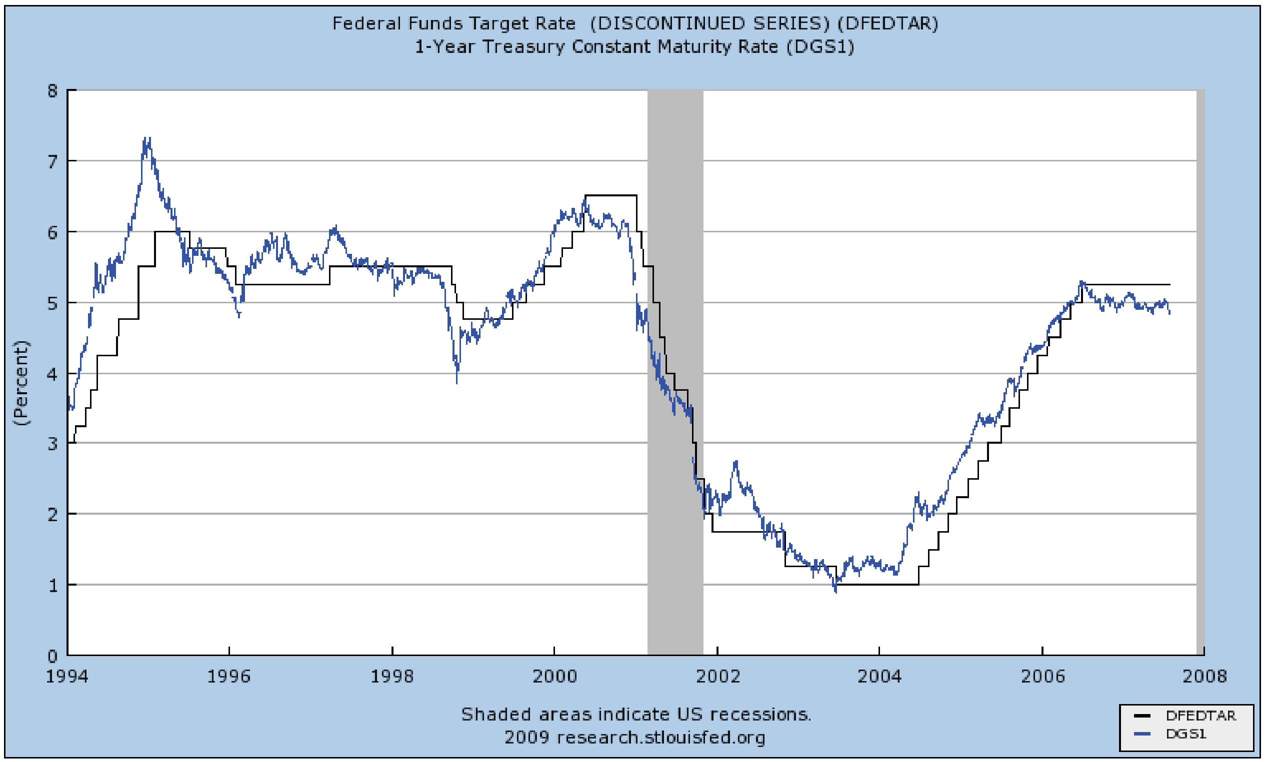

Table I: **LIBOR Rates Summary Statistics**

Means (μ_i) and standard deviations (σ_i) of simple annualized LIBOR rates (Panel (a)) and forward LIBOR rates (Panel (b)) for maturities from 1 to 12 months. Daily data from Jan. 3rd, 1994 to July 31th, 2007.

(a) LIBOR Rates

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
μ_{Lib}	4.33	4.37	4.41	4.44	4.47	4.50	4.54	4.57	4.60	4.63	4.67	4.70
σ_{Lib}	1.76	1.77	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.77	1.77	1.77

(b) Forward Rates

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
μ_{For}	4.33	4.41	4.49	4.54	4.61	4.66	4.73	4.79	4.84	4.95	5.01	5.07
σ_{For}	1.76	1.78	1.80	1.80	1.80	1.80	1.79	1.79	1.79	1.78	1.77	1.74

Table II: **Futures Rates Summary Statistics**

Means (μ_i) and standard deviations (σ_i) of futures rates (Panel (a)) and of the spreads between futures and forward rates (Panel (b)) for horizons of 1 to 6 months. Daily data from Jan. 3rd, 1994 to July 31th, 2007.

(a) Futures Rates

	1	**2**	**3**	**4**	**5**	**6**
μ_{Fut}	4.18	4.21	4.25	4.28	4.33	4.37
σ_{Fut}	1.74	1.75	1.75	1.76	1.76	1.75

(b) Forward-to-Futures Spreads

	1	**2**	**3**	**4**	**5**	**6**
μ_{diff}	0.14	0.20	0.24	0.25	0.28	0.29
σ_{diff}	0.14	0.15	0.18	0.17	0.17	0.14

B Summary Statistics

Table I presents summary statistics of LIBOR rates and LIBOR forward rates implicit from the LIBOR term structure. The average term structure of LIBOR rates is upward sloping, from 4.32% to 4.70%, implying a positive average term premium (Panel (a)). On the other hand, the term structure of LIBOR volatilities is almost flat, ranging from 1.76% to 1.78%, but with slight a hump-shape (1.80%) at intermediate maturities. Panel (b) presents statistics of LIBOR *forward* rates which are comparable to futures rates since they have the same reference period. The average term structure of forward rates is steeper than for LIBOR rates, 4.32% to 5.07%, and the hump shape in volatilities is more pronounced. Table II(a) presents summary statistics of futures rates. Futures rates averaged between 4.18% and 4.37% for maturities from 1 to 6 months with volatilities ranging from 1.74% and 1.76%. Table II(b) provides summary statistics for the spreads between LIBOR forward rates and futures rates. Clearly, futures rates are on average lower and less volatile than forward rates but their term structure is not as steep. Finally, forward-to-futures spreads display substantial variability.

Table III: **Principal Component Analysis of Forward-Futures Spreads**

Principal component analysis of forward-to-futures spreads at monthly maturities from 1 to 6 months. Each column displays the loading across maturities and the contribution to the variance explained for each component. Daily data from Jan. 3, 1994 to July 31, 2007 but excluding the last 9 months of 1999.

	Components Loadings					
Maturity	PC1	PC2	PC3	PC4	PC5	PC6
1	0.44	0.77	-0.05	-0.10	0.29	0.31
2	0.38	0.13	-0.02	0.14	-0.90	-0.00
3	0.42	0.04	0.08	0.24	0.23	-0.83
4	0.37	-0.28	0.18	-0.85	-0.02	-0.06
5	0.37	-0.32	0.65	0.37	0.15	0.39
6	0.43	-0.43	-0.72	0.16	0.16	0.21
R^2	0.53	0.13	0.12	0.09	0.07	0.06
$Cum.R^2$	0.53	0.66	0.77	0.88	0.94	1

Table IV: **Excess Returns and Components from Forward-to-Futures Spreads**

Results from predictive regressions of monthly futures excess returns (Panel (a)) and monthly forward excess returns (Panel (b)),

$$xr_{t,n} = \gamma_{0,n} + \gamma_{lvl}\ level_t + \gamma_{slp}\ slope_t + u_{t,n},$$

where $level_t$ and $slope_t$ are the first two components extracted from forward-to-futures spreads. Regressors are centered around zero and normalized by their standard deviations. Excess returns are in basis points (annualized). I include t-statistics based on Newey-West standard errors (6 lags) in parenthesis and R^2 in brackets. Monthly data from January 1994 to June 2007.

(a) Futures Excess Returns

	2	**3**	**4**	**5**	**6**
γ_0	0.02	0.02	0.03	0.03	0.04
	(2.35)	(1.99)	(2.14)	(1.99)	(1.94)
γ_{lvl}	0.00	-0.03	-0.02	-0.03	-0.02
	(0.15)	(-1.04)	(-1.05)	(-1.15)	(-1.03)
γ_{slp}	0.03	0.05	0.06	0.07	0.07
	(1.41)	(2.78)	(2.84)	(2.93)	(2.83)
R^2	[2.8]	[8.3]	[7.4]	[6.5]	[5.6]

(b) Forward Excess Returns

	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
γ_0	0.08	0.06	0.04	0.08	0.05	0.08	0.07	0.07	0.12	0.07	0.07
	(6.54)	(4.29)	(2.23)	(4.39)	(2.59)	(2.80)	(2.98)	(2.29)	(3.34)	(2.07)	(1.79)
γ_{lvl}	0.03	0.01	-0.02	0.01	-0.00	0.02	0.03	0.04	0.04	0.03	0.01
	(0.91)	(0.55)	(-0.63)	(0.41)	(-0.02)	(0.65)	(1.27)	(1.24)	(1.09)	(0.92)	(0.13)
γ_{slp}	0.05	0.05	0.07	0.09	0.10	0.11	0.10	0.09	0.14	0.11	0.07
	(2.37)	(2.14)	(2.84)	(3.18)	(3.55)	(3.04)	(2.86)	(2.73)	(3.39)	(2.58)	(1.75)
R^2	[5.7]	[3.5]	[6.6]	[5.8]	[7.3]	[6.4]	[4.8]	[4.1]	[5.4]	[4.3]	[1.5]

C The Information Content of Forward-to-Futures Spreads

Forward-to-futures spreads are volatile but share a factor structure across maturities. Table III displays results from a Principal Components Analysis.[3] The first two components explain 66% of the total variance and signal changes in the level and slope of spreads across maturities, respectively. This provides direct justification to model the term structure of forward-to-futures spreads in terms of a few underlying factors. More importantly, the slope component predicts excess returns. I compute excess returns from holding the futures contracts for calendar month n until the following month,

$$xr_{t,n}^{mth} = F(t,n) - F(t+m,n), \tag{1}$$

and use the level and slope components of forward-to-futures spreads in predictive regressions.[4] Table IV presents the results. The level is never significant, but the slope is significant for contract maturities of 3 months or more. Increasing the slope by one standard deviation increases expected monthly excess returns by 5 to 7 bps (annualized). This is economically significant relative to average excess returns of 2 and 4 bps. Table IV(b) shows similar results for excess returns on forward rates. Hence, forward-to-futures spreads predict excess returns and can be used to improve policy rate forecasts. This justifies combining forward and futures in a dynamic term structure model.

III Model

A Continuous versus Discrete Supports

The overnight federal funds rate is the main policy instrument used by the Federal Reserve to reach its long-term objectives. The FOMC determines a target for the overnight rate and, since 1994, its operational procedure features three salient properties. First, the FOMC announces publicly any decision about the target rate immediately following its meeting. Second, the committee publishes its meeting schedule well in advance and almost all target changes followed a scheduled meeting. Third, all changes effected by the FOMC to its target were in discrete increments of 25

[3] The analysis excludes the last 9 months of 1999 because of the effect of the Millenium change. See Section III.

[4] Returns observations do not overlap and standard asymptotic inference remains valid (see e.g. Richardson and Stock (1989), Valkanos (2002) and Diez de los Rios and Sentana (2010)). Moreover, the regressors' persistence cannot be driving the results along the lines suggested by Stambaugh (1999). Assuming that the level and slope factors follow AR(1) processes, estimates of the persistence coefficients are 0.66 and 0.50, respectively, and estimates of the correlations between AR(1) innovations and residuals from predictive regressions range between -0.10 and -0.35 for the level factor and -0.06 and 0.20 for the slope factor. On the other hand, the results are conservative since short-horizon returns are likely to be noisy relative to the ex-ante risk premium.

basis points [bps]. Therefore, the target rate follows a step-like path whereas changes almost always occur on pre-determined meeting dates and take a small number of values on a discrete support.[5] In contrast, a central assumption of most interest rate models is that the short rate evolves smoothly over a continuous support. As an illustrative example, a typical modeling strategy is to write the short rate as the sum of 3 continuous factors that evolve smoothly. That is,

$$\begin{aligned} r_t =& a + b_1 X_{1,t} + b_2 X_{2,t} + b_3 X_{3,t} \\ dX_t =& \kappa(\theta - X_t)dt + \Sigma dW_t \end{aligned}$$

where dW_t is a standard Brownian process. In this case, standard results show that yields are linear in the risk factors, X_t, and inherit their continuous support.

Figure 1 compares the target rate with the 1-year Constant Maturity Treasury yield. The target rate is a pure jump process but, nonetheless, the evolution of the 1-year yield is gradual and smooth. To see why this is the case, define the set of future dates, $\mathbb{S}_t$, with a scheduled meeting and write the one-year yield, $y_t^{(i)}$, in terms of target of rate expectations and a risk premium,

$$y_t^{(1)} = E_t \left[\frac{1}{365} \sum_{i=1}^{365} r_{t+i-1} \right] + rp_t^{(1)} = r_t + E_t \left[\sum_{i \in \mathbb{S}_t} c_i \Delta_{t+i} \right] + rp_t^{(1)}$$

where time is daily, $\Delta_{t+i} = r_{t+i} - r_{t+i-1}$, $c_i = \frac{365-i}{365}$ and $rp_t^{(1)}$ is an adjustment for risk. Yield changes are then given by,

$$y_{t+1}^{(1)} - y_t^{(1)} = \sum_{i \in \mathbb{S}} (E_{t+1} - E_t) [c_i \Delta_{t+i}] + rp_{t+1}^{(1)} - rp_t^{(1)}, \tag{2}$$

which shows that yields can change smoothly as economic information arrives since expectations and, a fortiori, expectation changes are not restricted to the discrete support. Averaging across future meetings smooths the evolution of yields further. Finally, risk premium variations can also evolve continuously. Then, a continuous support approximation may be sufficiently accurate for many research questions. But it is costly for others. In particular, it affects estimation of the monetary policy rule, of monetary shocks and of the risk premium from short-maturity yields at relatively high frequencies. Hamilton and Jordà (2002) and Grammig and Kehrle (2008) show that accounting for the discrete nature of target changes improves forecasts. Equation 2 implies that a more accurate description of future target changes, Δ_{t+i}, implies a more accurate description

[5] All three features are important. Previous to 1994, uncertainty as to the timing and magnitude of target changes caused continuous variations in investors predictions of the unobserved target rate as economic information arrived.

of the risk premium variations. Piazzesi (2005a) shows that a jump-only specification produces smaller LIBOR pricing errors relative to standard diffusion-based models and, therefore, yields a more accurate description of the risk-neutral dynamics.

B A Model of the Target

The above discussion suggests the following representation of target rate changes. Decompose the effective overnight rate, r_t^{eff}, into the Fed's target and a spread, $r_t^{eff} = r_t + s_t$. Then, target changes can be written as,

$$r_{t+1} - r_t = \Delta n_t,$$

where Δ is the 0.25% increment and n_t is distributed over a discrete integer support. Except for a discrete support, this representation is completely general without more restrictions on the properties of n_{t+1}. In practice, every FOMC meeting resulted in changes of less than 4 increments and most resulted in changes of 0 or 1 increment. A simple representation of these rare events is obtained based on the difference between two Poisson processes, n_{t+1}^u and n_{t+1}^d, representing up and down jumps, respectively,

$$r_{t+1} - r_t = \Delta \left(n_{t+1}^u - n_{t+1}^d \right), \tag{3}$$

where, conditional on time-t information, n_{t+1}^u and n_{t+1}^d are independent Poisson random variables with time-varying jump intensities, λ_{t+1}^u and λ_{t+1}^d, when there is a scheduled FOMC meeting and constant intensities λ_0^u and λ_0^d otherwise.[6]

The intensities are driven by state variables summarizing the economic information set relevant to the FOMC for monetary policy purposes. First, I assume that the FOMC considers the current target rate, r_t, when deciding on its future course. Second, the committee also observes a wealth of macroeconomic information that will be captured by a latent variable, z_{t+1}. Third, I introduce a separate latent variable, l_{t+1}, to capture the information from forward-to-futures spreads. In effect, this assumes that policy-makers consider interest rates from different markets. I discuss these latent variables below. Finally, I include the effective spread, s_{t+1}, among the state variables for the purpose of pricing fed funds futures. Then, before announcing the course from r_t to r_{t+1}, the committee observes lagged values of the target, r_t and contemporaneous values of z_{t+1} and l_{t+1}. Its information set *before* any policy decision is summarized by

$$X_{t+1}^* \equiv \left[r_t \ s_{t+1} \ z_{t+1} \ l_{t+1} \right]^T. \tag{4}$$

[6] The FOMC schedule is known in advance. Piazzesi (2005b) discusses jumps with time-varying intensities but with deterministic jump time.

In contrast, the information set of private agents pricing bonds and futures *following* a policy announcement[7] if any, is summarized by the vector X_{t+1},

$$X_{t+1} \equiv [r_{t+1}\ s_{t+1}\ z_{t+1}\ l_{t+1}]^T. \tag{5}$$

Finally, the jump intensities (at scheduled meetings) are affine functions of deviations between X_t^* and its mean $\bar{X} \equiv E[X_t^*]$,

$$\lambda_{t+1}^u = \lambda + \lambda_u^T \left(X_{t+1}^* - \bar{X}\right) \text{ and } \lambda_{t+1}^d = \lambda - \lambda_d^T \left(X_{t+1}^* - \bar{X}\right), \tag{6}$$

so that shocks to state variables affect upward and downward jumps in opposite directions.

C Properties of the Target Rate

Most monetary policy rules specify the relationship between economic variables and the conditional expectation of future policy rates. Clarida et al. (2000) provides a case in point. They study the following short rate specification (in their notation),

$$E_t[r_{t+1}] = (1-\rho)\left[r^* + \beta(\pi_{t+1} - \pi^*) + \gamma x_{t+1}\right] + \rho(L) r_t \tag{7}$$

where r^* is the long-run target for the overnight rate, $(\pi_t - \pi^*)$ is the inflation gap and x_t is the output gap. This contrasts with Equation 3 where the policy rule specifies a conditional distribution. Nonetheless, it is easy to show, using results for the Skellam distribution, that conditional on X_{t+1}^*, the expected target change is given by

$$E[r_{t+1} - r_t | X_{t+1}^*] = \Delta(\lambda_u + \lambda_d)^T (X_{t+1}^* - \bar{X}), \tag{8}$$

and varies linearly with anticipated deviations of state variables from their means. This not unlike Taylor-rule specifications but with different state variables and parametric restrictions. Note that λ_u and λ_d cannot be identified separately based on the conditional mean equation via regressions of target rate changes on state variables. On the other hand, these parameters also mediate changes of higher-order moments. The conditional variance of target rate changes is given by

$$Var_t[r_{t+1} - r_t | X_{t+1}^*] = \Delta^2(2\lambda + (\lambda_u - \lambda_d)^T (X_{t+1}^* - \bar{X})), \tag{9}$$

[7]This implies that prices used in the estimation must be observed following a policy announcement. Also, agents perceptions of the latent z_{t+1} and l_{t+1} must correspond closely to that of the FOMC at the time of the meeting. In other words, the intra-day variations following the announcement are small. Finally, including s_{t+1} in X_{t+1}^* is a slight abuse of notation but this is irrelevant for the results.

and also varies linearly with states variables. The sign of this relationship is given by the sign of $\lambda_u - \lambda_d$. To see this, suppose that the state of the economy is positively linked with z_{t+1} (e.g. z_{t+1} is a measure of employment) and that an increase of z_{t+1} raises the intensity of an up jump and decreases the intensity of a down jump. Then, $\lambda_{u,z}$ and $\lambda_{d,z}$ should be positive. If $\lambda_{u,z} > \lambda_{d,z}$ and economic conditions improve then the increase in the variance of positive jumps is greater than the decrease in the variance of negative jumps and, therefore, the total variance of target changes increases, all else equal. Moreover, the mean and variance of target changes are positively correlated. The converse holds if $\lambda_{u,z} < \lambda_{d,z}$.

Finally, the conditional skewness of target changes is given by

$$Skew[r_{t+1} - r_t | X^*_{t+1}] = (\lambda_u + \lambda_d)^T (X^*_{t+1} - \bar{X}) / ((2\lambda + (\lambda_u - \lambda_d)^T (X^*_{t+1} - \bar{X})))^{3/2}, \tag{10}$$

and always varies with state variables. The relationship is positive and very similar to that of the conditional mean whenever $\lambda_u = \lambda_d$ but, more generally, the sign depends on the relative magnitude of $\lambda_{u,z} + \lambda_{d,z}$ and $\lambda_{u,z} - \lambda_{d,z}$. If $\lambda_{u,z} < \lambda_{d,z}$ then skewness increases with improving economic conditions. The impact is ambiguous otherwise. In the stationary state where each state variable is equal to its unconditional mean, $X^*_{t+1} = \bar{X}$, the distribution of target rate changes is symmetric around zero with variance $2\lambda\Delta^2$.

D Historical Dynamics

Latent factors, z_{t+1} and l_{t+1}, follow a Gaussian VAR(1) dynamics,

$$\begin{bmatrix} z_{t+1} \\ l_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_z \\ \mu_l \end{bmatrix} + \begin{bmatrix} \phi_z & \phi_{z,l} \\ \phi_{l,z} & \phi_l \end{bmatrix} \begin{bmatrix} z_t \\ l_t \end{bmatrix} + \begin{bmatrix} \epsilon^z_{t+1} \\ \epsilon^l_{t+1} \end{bmatrix} \tag{11}$$

where ϵ_t are *i.i.d.* multivariate Gaussian innovations. The dynamics of the effective spread allows for its documented leptokurtic distribution,

$$s_{t+1} = \mu_s + \phi_s s_t + \epsilon^s_{t+1} + J^s_{t+1}, \tag{12}$$

with $\epsilon^s_t \sim N(0, \sigma_s)$. The jump term, J^s_{t+1}, follows a compound Poisson distribution with number of jumps $n^s_{t+1} \sim P(\lambda_s)$ and jump size $\nu^s_{t+1} \sim N(\nu_s, \omega^2_s)$, conditional on the number of jumps. The assumption that s_{t+1} does not respond to z_t and l_t is consistent with the Fed's explicit actions to counteract any predictable deviations of the effective rate from its target.

The state vector admits the following VAR representation,

$$X_{t+1} = \mu(I_{t+1}) + \Phi(I_{t+1})X_t + \xi_{t+1}, \tag{13}$$

where the indicator I_{t+1} is equal to 1 if a meeting is scheduled at date $t+1$ and zero otherwise.[8] The VAR parameters are given by,

$$\mu(0)^T = \begin{bmatrix}\mu_r(0) & \mu_s & \mu_z & \mu_l\end{bmatrix} \quad \text{and} \quad \mu(1)^T = \begin{bmatrix}\mu_r(1) & \mu_s & \mu_z & \mu_l\end{bmatrix}, \tag{14}$$

and

$$\Phi(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \phi_{s,r} & \phi_s & \phi_{s,z} & \phi_{s,l} \\ \phi_{z,r} & \phi_{z,s} & \phi_z & \phi_{z,l} \\ \phi_{l,r} & \phi_{l,s} & \phi l, z & \phi_l \end{bmatrix} \quad \text{and} \quad \Phi(1) = \begin{bmatrix} \phi_r(1) & \phi_{r,s}(1) & \phi_{r,z}(1) & \phi_{r,l}(1) \\ \phi_{s,r} & \phi_s & \phi_{s,z} & \phi_{s,l} \\ \phi_{z,r} & \phi_{z,s} & \phi_z & \phi_{z,l} \\ \phi_{l,r} & \phi_{l,s} & \phi l, z & \phi_l \end{bmatrix}, \tag{15}$$

Parameters of the target rate process, $\mu_r(I_{t+1})$, $\phi_r(I_{t+1})$, $\phi_{r,s}(I_{t+1})$, $\phi_{r,z}(I_{t+1})$ and $\phi_{r,l}(I_{t+1})$ are functions of primitive parameters and of the policy meeting schedule (See Equation 8). Moreover, the joint density of ξ_{t+1}, conditional on X_t, is known in closed-form and the system is amenable to estimation via Maximum Likelihood.

Although this is not the objective of this paper, the auto-regressive representation implies that the policy response function is applicable in the context of a structural VAR. The specification used here has $\phi_{s,r} = \phi_{z,r} = \phi_{l,r} = 0$ for simplicity since I do not attempt to identify the underlying structural shocks. However, in the general case where those parameters are unrestricted, a common identifying assumption to recover the structural shock is that monetary policy shocks are uncorrelated with macro-economic variable shocks. This is precisely one of the main assumptions in Section III. Innovations in any of the state variables affect r_{t+1} only through their structural impact on the jump intensities, λ^u_{t+1} and λ^d_{t+1}, via the sensitivity parameters λ_u and λ_d on FMOC days only.[9]

[8]This follows since the joint conditional Laplace transform is exponential-affine. It would belong in the CAR family of Darolles et al. (2006) if we eliminated the deterministic dependence on the meeting schedule. For example, we can use the FOMC meeting schedule to define the sampling frequency relevant for the purposes of monetary analysis so that $I_{t+1} = 1 \ \ \forall t$.

[9]Another way to see the implicit structural restriction is by approximating ξ_{t+1} with $\Sigma(I_{t+1})u_{t+1}$ where u_{t+1} are uncorrelated white noises and the matrix $\Sigma(I_{t+1})$ is of the form,

$$\Sigma(I_{t+1}) = \begin{bmatrix} \sigma^2_r(I_{t+1}) & \sigma_{r,Y}(I_{t+1}) \\ 0 & \Sigma_Y \end{bmatrix}.$$

and $\sigma_{r,Y}(I_{t+1})$ depends on primitive parameters.

E *Risk-Neutral Dynamics*

The risk-neutral measure is not unique since markets are incomplete. I assume that the stochastic discount factor [SDF], M_{t+1}, is exponential-affine,

$$M_{t+1} = \exp\left(-r_t^{eff}\right) \frac{\exp(\delta_t^T X_{t+1})}{E_t\left[\exp\left(\delta_t^T X_{t+1}\right)\right]}, \tag{16}$$

where δ_t is the vector of prices of risk. Gourieroux and Monfort (2007) show that this class nests a wide range of equilibrium-based SDFs. I assume that prices of risk are affine in the state variables,

$$\delta_t = \delta_0 + \delta_1 X_t, \tag{17}$$

with δ_0 a $K \times 1$ vector and δ_1 a $K \times K$ matrix. Clearly, time-varying prices of risk induces a time-varying risk premium. However, the risk premium may vary even if prices of risk are constant if the distribution of the target rate is time-varying (i.e. $\lambda_u \neq \lambda_d$).[10] Hence, I keep the price of target rate risk constant to disentangle the effect of varying quantities of risk on the risk premium from the effect of varying the price of target rate risk. I also keep constant the price of risk associated with l_t to enforce the identification of that factor with the risk premium implicit in forward-to-futures spreads. Otherwise, innovations in l_t could induce an additional risk premium. Instead, I attribute all price of risk variations to innovations in economic conditions, z_t. Finally, s_t has a constant price of risk. These restrictions maintain parsimony and ensure that the results can be interpreted easily.

Proposition 1 provides the solution of the price-generating function, $\Gamma(u, t, m)$, obtained from computing the time-t price of the generating payoff, $\exp(u^T X_{t+m})$. Asset prices can be derived from $\Gamma(u, t, m)$ (Duffie et al., 2000). When $m = 1$, $\Gamma(u, t, 1)$, corresponds to the joint conditional Laplace transform of X_{t+1} under $\mathbb{Q}$. Coefficients of this transform are of the same form as the corresponding coefficients under $\mathbb{P}$ and, therefore, X_{t+1} belongs to the same (conditional) distribution under both measures. Moreover, the shift between risk-neutral and historical parameters is given by

$$\begin{aligned} \mu^Q(0) &= \mu(0) + \Sigma\delta_0 \qquad & \mu^Q(1) &= \mu(1) + \Sigma(\delta_0 + G(\delta_{0,r})) \\ \Phi^Q(0) &= \Phi(0) + \Sigma\delta_1 \qquad & \Phi^Q(1) &= \Phi(1) + \Sigma\delta_1, \end{aligned} \tag{20}$$

where $G(\delta_{0,r})$ is defined in the Appendix. In particular, the target rate is conditionally Skellam

[10] See Polimenis (2006) for a discussion of this effect in economies with power utilities.

Proposition 1 Price Generating Function

In the absence of arbitrage opportunities, the time-t discounted value of the generating payoff $\exp(u^T X_{t+m})$ *at time* $t+m$ *is given by the price-generating function,* $\Gamma(u,t,m)$,

$$\begin{aligned}\Gamma(u,t,m) &= E_t\left[M_{t,t+m}\exp\left(u^T X_{t+m}\right)\right]\\ &= E_t\left[M_{t+1}\Gamma(u,t+1,m-1)\right],\end{aligned}$$

for $u \in \mathbb{R}^4$, *where* $M_{t,t+i} \equiv M_{t+1}\cdots M_{t+i}$, $M_{t,t+1} = M_{t+1}$ *and by convention* $M_{t,t} = 1$. *The price-generating function has the following exponential-affine solution,*

$$\Gamma(u,t,m) = \exp\left(c_0(u,t,m) + c(u,t,m)^T X_t\right), \tag{18}$$

where coefficients satisfy,

$$\begin{aligned}c_0(u,t,m) &= c_0(u,t+1,m-1) + A^Q(I_{t+1}, c(u,t+1,m-1)))\\ c(u,t,m) &= B^Q(I_{t+1}, c(u,t+1,m-1)),\end{aligned} \tag{19}$$

and I_t *is an indicator function equal to 1 if an FOMC meeting is scheduled at* $t+1$ *and 0 otherwise. Initial conditions,* $c_0(u,t,0) = 0$ *and* $c(u,t,0) = u$, *are given by* $\Gamma(u,t,0) = \exp(u^T X_t)$ *(see appendix C).*

---

with intensity parameters given by

$$\begin{aligned}\lambda^Q_{0,u} &= \lambda_0 \exp(\delta_{0,r}\Delta) & \lambda^Q_{0,d} &= \lambda_0 \exp(-\delta_{0,r}\Delta),\\ \lambda^Q_{u} &= \lambda_u \exp(\delta_{0,r}\Delta) & \lambda^Q_{d} &= \lambda_d \exp(-\delta_{0,r}\Delta).\end{aligned} \tag{21}$$

Note that the policy function is not symmetric under the risk-neutral measure (i.e. $\lambda^Q_u \neq \lambda^Q_d$ and $\lambda^Q_{0,u} \neq \lambda^Q_{0,d}$) whenever target rate risk is priced (i.e. $\delta_{0,r} \neq 0$) even if the policy function is symmetric under $\mathbb{P}$. Therefore, the model can generate stochastic volatility in yields, and a variance spread, even if the target rate has constant volatility under $\mathbb{P}$. Finally, another important implication is that the cross-section of yields is informative about parameters of the policy function under $\mathbb{P}$ (i.e. λ_u and λ_d). This contrasts with dynamic Gaussian term structure models where imposing the no-arbitrage restrictions does not affect $\mathbb{P}$-measure conditional expectations (Joslin et al. (2010)).

F LIBOR Loans

The price of a risk-free discount bond is obtained from the price-generating function by setting $u = 0$. On the other hand, LIBOR loans are short-term unsecured inter-bank loans and investors require an extra yield to hold them. Specifically, I assume that the price, $D^l(t,m)$, of a LIBOR loan with maturity m is given by,

$$D^l(t,m) \equiv E_t\left[M_{t,t+m}\exp\left(-\sum_{i=0}^{m-1} l_{t+i}\right)\right] = \exp\left(d_0^L(t,m) + d^L(t,m)^T X_t\right), \tag{22}$$

with coefficients given in the Appendix. The interpretation of l_t is best seen when comparing with the price of a risk-free loan, $D^{rf}(t,m) \equiv E_t\left[M_{t,t+m}\right]$. This makes clear that l_t represents the marginal compensation offered by LIBOR loans. This premium may represent a compensation for default risk. It may also be due to a combination of market power, barriers to entry, or capital-constrained arbitrageurs. This reduced-form approach borrows from Grinblatt (2003) and Duffie and Singleton (1997).

Drossos and Hilton (2000) document the impact of Y2K fears on funding rates (see also Sundaresan and Wang (2009)). Figure 2 compares futures and forward LIBOR rates. It shows that LIBOR rates on loans initiated in 1999 jumped upward when their maturity first extended into 2000. Banks placed a greater value on being liquid or charged more for counterparty risk on the last day of the 20th century. A dummy variable is introduced to capture this effect. The Millennium premium, l^*, is defined through the following modification of the LIBOR loan equation,

$$D^L(t,m) = E_t\left[M_{t,t+m}\exp\left(-\sum_{i=0}^{m-1} l_{t+i} + l^* I(t+i,m)\right)\right].$$

where t^* is the last business day of 1999 (i.e. Dec. 28) and the indicator $I(t,m)$ is equal to one if $t < t^* \le t+m$ and zero otherwise.

G Federal Funds Futures

A futures contract settles at the end of a reference calendar month, n. Its payoff is the difference between the contract rate and the average overnight fed funds rate in the reference month, $\bar{r}_n$. With no loss of generality, I standardize the notional of the contract to 1. Futures contracts require no investment at inception. A futures rate, $F(t,n)$, is equal to the discounted value of its payoff,

$$\begin{aligned} F(t,n) &= E_t\left[M_{t,t+T_n^*}\bar{r}_n\right] = E_t\left[M_{t,t+T_n^*} D_n^{-1} \sum_{i=T_n-D_n}^{T_n} r_{t+i}\right] \\ &= D_n^{-1} \sum_{i=T_n-D_n}^{T_n} E_t\left[M_{t,t+T^*} r_{t+i}\right] = D_n^{-1} \sum_{i=T_n-D_n}^{T_n} f(t,i,T_n^*), \end{aligned} \tag{25}$$

Proposition 2 Price of a Singleton futures Contract

The rate, at time-t, of a singleton futures contract, $f(t,h,T)$, for the reference day $t+h$ and settling at date $t+T$ is

$$\begin{aligned} f(t,h,T) \equiv& E_t\left[M_{t,t+T} r_{t+h}\right], \\ =& \frac{\partial}{\partial u} E_t\left[M_{t,t+T}\exp(u r_{t+h})\right]\Big|_{u=0}, \;\; 0 \le h \le T, \\ f(t,h,T) =& \exp\left(d_0^{rf}(t+h,T-h) + c_0\left(u^*,t,h\right)\right) + c\left(u^*,t,h\right)^T X_t\Big) \\ & \times \left[c_0^{'}\left(u^*,t,h\right)C_r + X_t^T c^{'}\left(u^*,t,h\right)C_r\right], \end{aligned} \qquad (23),(24)$$

where $u^* = d(t+h,T-h)$ and $C_r = [1\ 1\ 0\ 0]^T$. The coefficients $c_0^{'}(\cdot)$ and $c^{'}(\cdot)$ satisfy recursions given by the derivatives with respect to u of Equation (19). Initial conditions are obtained by noting that $f(t,0,T) = D^{rf}(t,T)r_t$ which implies that $c_0^{'}(u^*,t,0) = 0$ and $c^{'}(u^*,t,0) = I$ for any t (see Appendix C).

where T_n is the number of days between t and the end of the reference month and D_n is the number of days in that month.[11] Due to weekends or holidays, the settlement date, $t+T_n^*$, may not coincide with the last day of the month, $t+T_n$. The price of a singleton futures contract, $f(t,i,T_n^*)$, in Equation 25, is given explicity in Proposition 2 where $T^* = T$ to simplify the presentation.

IV Estimation Results

Equation 13 provides the transition equation for a state-space representation of the model. The measurement vector, $\tilde{Y}_t = [r_t\ s_t\ Y_t']'$, includes the target rate, the effective spread as well as LIBOR rates and futures rates, stacked in vector Y_t. Assuming i.i.d zero-mean and uncorrelated Gaussian measurement errors, estimation is conducted via Quasi Maximum Likelihood [QML] using the Unscented Kalman Filter [UKF] to account for non-linearities due to futures rates. Details of the estimation and filtering algorithms are provided in Appendix A. Briefly, conditional on values of the latent state variables, the joint conditional likelihood is known in closed-form. The UKF is an approximate filter that only matches the first two moments of the state distribution and a QML

[11] The quoted price of this contract, $P(t,n)$, is given by $P(t,n) = 100 - F(t,n) \times 3600$

estimator is feasible. The joint log-likelihood is given by,

$$
\begin{aligned}
L(\Theta; Y) &= \sum_{t=1}^{T} \log\left(f_t\left(\tilde{Y}_t|\tilde{Y}_{t-1};\theta\right)\right) \\
&= \sum_{t=1}^{T} \log\left(f_t(Y_t|\hat{z}_{t|t-1}, \hat{l}_{t|t-1}, r_t, s_t) f(r_t|r_{t-1}, \hat{z}_{t|t-1}, \hat{l}_{t|t-1}, I_t) f(s_t|s_{t-1})\right)
\end{aligned}
$$

where all model parameters are grouped in the vector Θ. The conditional likelihood of Y_t depends on t through the deterministic FOMC schedule. I assume that $\lambda_{u,s} = \lambda_{d,s} = 0$ and that the Fed does not respond to the current spread in its evaluation of the appropriate policy stance. This implies the separation of the marginal likelihood of s_t and its dynamics can be estimated separately. Finally, the target rate reverses to a well-defined unconditional mean, $\bar{r}$, when all but one of the eigenvalues of $\Phi(I_{t+1} = 0)$ lie inside the unit circle and if all the eigenvalues of $\Phi(I_{t+1} = 1)$ lie inside the unit circle. Intuitively, if s_t, z_t and l_t are jointly stationary then mean-reversion follows if the intensity of an "up" change decreases and the intensity of a "down" change increases when the target rate increases.

A *Pricing Errors*

Table V presents parameter estimates. Table VI presents Mean Pricing Errors [MPE] and Root Mean Squared Pricing Errors [RMSPE] in Panel (a) and Panel (b), respectively. Pricing errors are low by any standard and the model provides a good fit of LIBOR and futures rates, improving upon the extant literature. Average errors are typically less than 1 bp and often less than one-tenth of 1 bp (annualized) across maturities. LIBOR RMSPE is 4.3 bps across all maturities and ranges from 10 bps at the shortest maturity to less than 1 bps at maturities of 6 months and beyond. Piazzesi (2005a) reports average absolute errors of 12.5, 7.5 and 6.8 bps for LIBOR rates at maturities of 1, 3 and 12 months. The corresponding figures, 6, 4 and less than 1 bps, are significantly lower in this longer sample. Futures RMSPE is 8.8 bps overall and increases with maturity, from 3 to 12 bps. Finally, estimates of pricing error variance parameters (unreported) are consistent with sample RMSPEs.

B *Forecasting Target Rates*

This section compares model-implied forecasts of future target rates with predictive regressions based on market rates. Results show that a parametric model adapted to the empirical properties of the target rate captures the information content of futures and forward rates and delivers accurate target rate forecasts. In fact, conditional expectations computed from the model provide unbiased

Table V: **Parameter Estimates**

Parameter estimates from daily data (January 1994 to July 2007). Panel (a) and (b) displays parameters for the latent variables and effective spread dynamics, respectively. Panel (c) displays parameter estimates for the policy function. Panel (d) displays price of risk parameters. In each case, standard errors are provided in parenthesis. The estimate for λ is 0.3293 and its standard error is (0.0054). In panel (a), the symbols * and ** indicates statistical significance at the 5% and 1% level, respectively, corresponding to the Wald statistics that individual coefficient are different from zero. (See Appendix A.4 for details.)

(a) Latent Factors Dynamics

	μ	$\phi_{i,i}$	$\phi_{i,j}$	σ	$\rho_{l,z}$
z_t	0	0.999^{**}	$1.89 \times 10^{-3**}$	7.91^{**}	-0.11^{**}
l_t	$1.03 \times 10^{-3**}$	0.992^{**}	-2.18×10^{-6}	$8.14 \times 10^{-2**}$	-

(b) Effective Spread Dynamics

	$\mu \times 10^{-3}$	ϕ	σ	ν_s	ω_s	λ_s
s_t	-3.54	0.20	0.05	0.69	0.27	0.23
	(0.53)	(0.012)	(0.001)	(0.005)	(0.013)	(0.022)

(c) Policy Function

	$r \times 10^3$	s	$z \times 10^{-3}$	l
λ_u	-104.04	0	4.63	9.05
	(0.055)	-	(0.034)	(0.069)
λ_d	-2.101	0	9.14	8.50
	(0.060)	-	(0.089)	(0.058)

(d) Prices Of Risk

	$r \times 10^3$	$s \times 10^3$	$z \times 10^{-3}$	$l \times 10^{-2}$
δ	-7.47	8.90	7.24	4.42
	(0.078)	(0.069)	(0.056)	(0.093)
	$r \times 10^{-2}$	s	$z \times 10^{-6}$	$l \times 10^{-3}$
$\delta_{1,z}$	-2.52	0	-2.91	6.25
	(0.059)	-	(0.080)	(0.054)

Table VI: **Pricing Error Statistics**

Mean Pricing Errors [MPE] and Root Mean Squared Pricing Errors [RMSPE]. The sample ranges from January 1994 to July 2007. Results are reported in percentage (annualized), for LIBOR rates and futures rates at maturities of 1 to 12 months and 1 to 6 months, respectively.

(a) Mean Pricing Errors ($bps \times 10^{-2}$)

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
LIBOR	-1.35	-0.48	0.27	0.12	0.19	0.05	0.03	-0.03	-0.20	-0.02	0.07	-0.08
Futures	-0.09	0.06	0.23	0.55	0.54	0.09						

(b) Root Mean Square Pricing Errors (bps)

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
LIBOR	0.10	0.07	0.06	0.04	0.03	0.01	0.01	0.01	0.01	0.01	0.00	0.01
Futures	0.03	0.06	0.08	0.10	0.11	0.12						

forecasts and are more accurate than benchmark regressions at horizons beyond 6 months. Clearly, investors and policy-makers benefit significantly from unbiased and accurate forecasts. This is also a key diagnostic check since the impetus behind modern dynamic term structure models was the presence of predictable and cyclical errors in forecasts based on the Expectation Hypothesis. The results show that the model provides a significant contribution in these directions. This was not preordained: delivering accurate target rate forecasts remains a challenge to standard term structure models.

Target rate forecasts based on the model are driven by estimates of the latent factor dynamics, given in Table V(a), and estimates of the Fed's policy function parameters, given in Table V(c). I discuss the latent factors below. The Fed's expected response to the current target is given by $\hat{\lambda}_{r,d} + \hat{\lambda}_{r,u}$ (see Equation 8). Both estimates are negative, highly significant and induces reversion of the target rate toward it long-term mean. The response to the macro factor, $\hat{\lambda}_{z,d} + \hat{\lambda}_{z,u}$, is highly significant and non-negative by assumption. The response to the liquidity factor is also positive and significant.[12] Based on these parameters, a time-t forecast of the effective overnight rate at date $t + h$ is given by,

$$E[r_{t+h} + s_{t+h}|X_t] = a_{r+s}(I_{t+1}, h) + b_{r+s}(I_{t+1}, h)^T X_t, \tag{26}$$

[12]These estimates only imply that target changes are positively correlated with each of z_t and l_t and do not pin down the direction of the structural relationship. Does the FOMC respond to innovations in these factors or vice-versa? This is beyond the scope of this paper but note that a structural analysis is feasible along the lines sketched out in Section D.

with coefficients $a_{r+s}(\cdot)$ and $b_{r+s}(\cdot)$ given in Appendix F.2. For comparability with existing results, I focus on forecasts of the average rate in calendar month n. The predicted average overnight rate for calendar month, $\bar{r}_n$, is given by,

$$\begin{aligned} E[\bar{r}_n|X_t] &= D_n^{-1} \sum_{i=1}^{D_n} a_{r+s}(I_{t+i}, h+i) + b_{r+s}(I_{t+i}, h+i)^T X_t, \\ &= \bar{a}(t,n) + \bar{b}(t,n)^T X_t, \end{aligned} \tag{27}$$

where D_n is the number of days in month n and h is the forecasting horizon between date t and the beginning of month n. Coefficients $\bar{a}(t,n)$ and $\bar{b}(t,n)$ depend on time through the deterministic variations in the FOMC meeting schedule and the number of days until the beginning of that calendar month.

As benchmark models, I use predictive regressions based on futures and LIBOR forward rates. Krueger and Kuttner (1996) use futures rates to forecast target rates. Gurkaynak et al. (2007) show that futures deliver the best (lowest RMSE) market-based forecasts of $\bar{r}_n$ up to 6-month ahead. Similarly, LIBOR rates deliver the best forecasts at longer horizons, up to a year. Recently Chun (2010) shows that futures deliver the best forecasts relative to surveys of professional forecasters and time-series models. Competing forecasts are given by,

$$\begin{aligned} E[\bar{r}_n|X_t] =& \bar{a}(t,n) + \bar{b}(t,n)^T X_t \\ E^{fut}[\bar{r}_n|X_t] =& \alpha_h^{fut} + \beta_h^{fut} F(t,n) \\ E^{lib}[\bar{r}_n|X_t] =& \alpha_h^{lib} + \beta_h^{lib} F_{lib}(t,n), \end{aligned} \tag{28}$$

where $F(t,n)$ and $F_{lib}(t,n)$ are the futures and forward LIBOR rates for calendar month n. Coefficients $\bar{a}(t,n)$ and $\bar{b}(t,n)$ are obtained directly from the estimation results above. Coefficients of the predictive regressions can be estimated via OLS. For each horizon, h, the time-series of monthly averages, $\bar{r}_n$, is regressed on a forward or futures rate observed h days before the beginning of each month.[13] I consider horizons $h = 1, 2, \ldots, 178$ when using futures rates and $h = 1, 2, \ldots, 360$ when using forward rates. There are no overlapping observations. Note that the resulting estimates directly minimize forecasting errors for each horizon separately and may suffer from in-sample over-fitting. On the other hand, predictive coefficients from the term structure model are computed from primitive parameters obtained from fitting the cross-section of futures and forward

[13] Model-based and futures-based forecasts exactly match the calendar month for any horizon h. On the other hand, the reference periods underlying forward rates have small variations in overlap for different h. This induces small RMSE variations across horizons. However, the overlap is constant for a given horizon (regression) and the match is exact for a subset of horizons.

rates. This mitigates the impact of over-fitting on forecasts.

Figure 3(a) compares forecast RMSEs. Figure 3(b) compares the corresponding R^2s. I also report results from random walk forecasts. Every predictor improves RMSE substantially relative to random walk forecasts. Futures outperform other forecasts at short horizons. At an horizon of 50 days, RMSEs are 10bps for futures-based forecasts and 15bps in the case of forward or model-based forecasts. In all cases, R^2s are very close to one. However, the advantage of futures deteriorates at longer horizons. At an horizon of 6 months, RMSEs are slighly above 45 bps for each predictor. At longer horizons, model-based forecasts provide improvements over forward-based forecasts. One-year ahead, R^2s are 63% and 60%, respectively.[14] Finally, forecasts from the model are unbiased. Average forecasting errors range between -2.4 and 2.7 bps across horizons and the associated t-statistics for the null of unbiased forecasts, accounting for autocorrelation up to 12 lags, always are far from the usual significance levels. The only exceptions are for horizons of a few days where economically tiny average errors, around 0.2 bps, appear statistically significant.

Figure 4 compares the time-series of forecasting errors. Each day, I compute the average forecasting errors across a range of horizons to remove some of the noise due to the daily idiosyncratic variations in individual futures and forward rates. Panel 4(a) compares errors over horizons between 3 and 6 months. Consistent with the results above, futures offer small benefits at these horizons. Panel 4(b) covers horizons between 6 and 9 months and Panel 4(c) covers horizons between 9 months and 1 year. Any forecasting model is bound to produce cyclical errors at these horizons since some of the target rate changes were truly unpredictable six months or one year in advance. Nonetheless, model-based forecasts are generally lower and less cyclical. The mean absolute errors of model forecasts is 17 bps lower than forward-based forecasts at horizons between 6 and 9 months and 22 bps lower at horizons between 9 and 12 months. This is the same magnitude as the typical target rate change.

C Risk Premium

The excess returns obtained from holding a futures contract referencing month n from time-t until its maturity are given by,

$$xr_{t,n} = F(t,n) - \bar{r}_n = F(t,n) - E_t[\bar{r}_n] - \epsilon_{t,n}, \tag{29}$$

[14] Results using futures and forwards are comparable with those of Gurkaynak et al. (2007) but they use a shorter sample period, ending in 2004, and a longer, quarterly, sampling frequency for horizons beyond 6 months.

where I use the $xr_{t,n}$ notation for excess returns.[15] The second equality shows that excess returns can be decomposed into a premium for risk, $F(t,n) - E_t[\bar{r}_n]$, and the forecast error, $\epsilon_{t,n}$. Returns tend to be higher when the risk premium is higher. But returns are also higher when target rate realizations are lower than expected (i.e. $\epsilon_{t+n} < 0$). This occurs either when the target rises less than anticipated in a tightening cycle or when it declines more than anticipated in a loosening cycle. The reverse is true for negative returns which occur when the target rises more than anticipated in a tightening cycle or when it declines less than anticipated in a loosening cycle. Hence it is hard to disentangle forecast errors from risk premium using regressions with only a small number of business cycles in the sample. On the other hand, a no-arbitrage model uses the cross-section of rates and can disentangle the risk premium from forecast errors. Empirically, the model presented here provides an accurate description of both components in Equation 29. It delivers low pricing errors and captures properties of the risk-neutral dynamics. It also provides accurate and unbiased target rate forecasts and captures properties of the historical dynamics. Therefore, it correctly measures the risk adjustment that arises in the passage from the historical to the the risk-neutral measure.[16]

Figure 8(a) compares the conditional risk premium, $E_t[xr_{t,n}]$ with ex-post realized returns from holding the 6-month ahead contract. The difference between returns and the risk premium is $\epsilon_{t,n}$: the unexpected component in the realization of $\bar{r}_n$. Realized returns are often in line with the risk premium but some large deviations occur, revealing the extent to which events were unforeseen. Some events are associated with financial shocks. For example, large excess returns toward the end of 1994 can be attributed to the Mexican Pesos crisis contributing to an early pause of the Fed's tightening cycle and to the subsequent target rate cuts. Also, large returns in 1998 can be attributed to the unforeseen collapse of Long Term Capital Management, its impact on financial stability, and the subsequent response by the Fed. Similarly, large returns observed toward the end of our sample followed the Fed's response to the initial stages of the 2007-2009 financial crisis. On the other hand, other deviations between the risk premium and returns appear to be associated with shocks in the real economy. For example, unforeseen economic developments might have caused some surprises in the magnitude or the speed of target cuts during the 2001 recession, especially following the terrorist attacks of September 2001. Conversely, the low returns in 2004 and 2006 may be due to surprises in the magnitude or timing of target rate hikes.[17]

[15] The initial investment required to hold a futures position is zero and its payoff is the difference between the initial futures rate and the realized monthly average effective rate in the settlement month. Piazzesi and Swanson (2006) show that accounting for variation margins due to marking positions to market prices produces near-identical excess returns (see the working paper version of their article).

[16] Ferrero and Nobili (2008) use surveys of professional forecasters to compute the ex-ante premium and ex-post forecast errors. However these forecasts appear biased (see their Table 4).

[17] Hamilton and Okimoto (2010) identifies two regimes in futures excess returns. Unsurprisingly, the episodes

D The Price of Macro Risk

The risk premium contribution to excess returns depends on the price of risk parameters, given in Table V(d). The price of z_t risk is positive on average implying that unexpected declines of this factor are associated with high marginal utility states, on average. All the components of $\delta_{1,z}$ are statistically significant and the price of z_t risk plays an important role in risk premium variations. Economically, z_t and l_t are the important drivers behind variations of $\delta_{z,t}$. The price of risk increases when z_t itself decreases, which is also consistent with the interpretation that a decline in z_t correspond to deteriorations of economic conditions. The price of macro risk also increases when forward-to-futures spreads increase. This implies that l_t affects futures and LIBOR rates indirectly via the risk premium. Finally, the price of macroeconomic risk also increases when the target rate is lower but this effect is small.

Estimates of the latent macro factor dynamics are not instructive. Instead, I check the interpretation of the macro factor by inspecting its relationships with other macroeconomic indicators. Figure 5(a) compares the macro factor with the target rate. The macro factor leads the policy rate and reveals investor forecasts of future economic conditions relevant to the FOMC when determining its target. The Fed's dual mandate suggests that the macro factor aggregates information about real activity and inflation. Figure 5(b) compares the macro factor with the Aruoba-Diebold-Scotti [ADS] index of U.S. real activity.[18] The relationship is visually apparent but the lead-lag relationships varies through the sample. Changes in the macro factor typically lead but sometime coincide with or lag the ADS index. The pattern suggests that the Fed's response to economic activity depends on the state of inflation. Absent a daily inflation index analog to the ADS index, the following case study provides a clear example that inflation is the key conditioning variable.

The ADS index declined sharply in 2000 and leading into the 2001 recession. Early in that year, the macro factor initially lagged the index because the FOMC emphasized inflation risk. In fact, the target rate rose to a high of 6.5% at the May 16th FOMC meeting. As late as November 15th, 2000 the FOMC statement noted that "softening in business and household demand and tightening conditions in financial markets over recent months suggest that the economy could expand for a time at a pace below [...] its potential to produce" but nonetheless concluded that "the risks continue to be weighted mainly toward conditions that may generate heightened inflation pressures in the foreseeable future." In its December 19th, 2000 statement, the committee noted that "rising

discussed here correspond to periods of high probabilities a regime with higher mean and volatility of returns.

[18] The Philadelphia Fed publishes the index daily and describes it as "designed to track real business conditions at high frequency. Its underlying economic indicators (Weekly initial jobless claims; monthly payroll employment, industrial production, personal income less transfer payments, manufacturing and trade sales; and quarterly real GDP) blend high- and low-frequency information and stock and flow data".

energy costs, as well as eroding consumer confidence, reports of substantial shortfalls in sales and earnings, and stress in some segments of the financial markets suggest that economic growth may be slowing further" but did not change its target rate. However, the committee shifted its risk assessment and stated that "the risks are weighted mainly toward conditions that may generate economic weakness in the foreseeable future". The macro factor had declined substantially by then and the market had largely anticipated this change of tone.[19] On January 3rd, 2001, the FOMC finally lowered its target for the federal funds rate by 50 bps following an unscheduled conference call. These events show that the macro factor correctly measured overall economic conditions, as perceived by the FOMC. Declining real activity, as measured by the ADS index, was weighted against the evolution of inflation. The period of rising interest rates in 2003-2004 offers another case. The FOMC delayed its response to improving economic conditions, citing undesirably low inflation, and the macro factor again lagged the ADS index for some time.

E Time-Varying Uncertainty

The Fed's anticipated response to economic conditions induces changes in expected target rates. This section shows that the Fed's response also induces cyclical changes in the uncertainty surrounding future target rates. This is a novel empirical result. The evidence suggests that the Fed's intents were less uncertain to market participants when economic conditions were good or improving, as measured by the macro factor. Conversely, poor or deteriorating economic conditions were associated with greater uncertainty regarding the Fed's future actions. In other words, market participants are more confident about the size and direction of policy changes, and the Fed's is perceived to act more rapidly and with less uncertainty, when leaning against inflation risk in a growing economy than when supporting employment in a slowing economy.[20] This is relevant for market participants since greater uncertainty in poor economic conditions increases the risk associated with exposures to target changes. Ultimately, this contributes to risk premium variations and adds to the difficulty of forecasting target rates based on market rates. It is also relevant to policy-makers since their behaviour may at times affect this uncertainty and reduce variation in the risk premium.

The uncertainty surrounding target rates varies with state variables because $\lambda_u \neq \lambda_d$. These changes are mostly driven by the target rate and by the macro factor but not the liquidity factor since $\lambda_{r,d} \gg \lambda_{r,u}$ and $\lambda_{z,d} \gg \lambda_{l,u}$ but $\lambda_{l,d} \approx \lambda_{l,u}$ (see Equation 9). Empirically, the variance of

[19]Interestingly, the macro factor reached a low point on the meeting's day and rose in the following days and weeks as the FOMC cut its target. The macro factor rose when the announcement made certain the coming expansionary policy and the associated effect on the economy.

[20]Economic conditions may also respond asymmetrically to monetary policy. See e.g. Garcia and Schaller (1999) and, more recently, Lo and Piger (2005).

target rate changes is lower (higher) when the target rate is high (low) *all else equal*. Similarly, the variance is lower (higher) when the macro factor is high (low), *all else equal*. This implies that volatility will be lowest when the target rate and the macro factor are high and, conversely, volatility will be highest when the target rate and the macro factor are low. I use a split-sample approach to analyze the magnitudes of the effects. I first split the sample using terciles of the macro factor and further divide each sub-sample using terciles of the target rate computed in each sub-sample. This produces 9 sub-samples which account for the observed comovements of the macro factor and of the target rate. For example, the first sub-samples (i.e. *(Low, Low)*, *(Low, Medium)*, *(Low, High)*) vary the target rate tercile from low values to high values, conditional on the macro factor being in its lowest tercile.

Figure 6 reports the conditional volatility, skewness and kurtosis computed for the average value of state variables in each sub-sample. Each line corresponds to a fixed value of the macro factor but different values of the target rate. Panel (a) displays the conditional volatility. The results are striking. Volatility is highest when the macro factor is low ranging from 30 % for low target rates to 25 % for high target rates in that tertile. It declines to around 20% and 10 % for medium and low values of the macro factor, respectively. Panel (b) displays the conditional skewness of target changes. Skewness is high and positive, between 4 and 5.25, when the macro factor is in its top tercile (but volatility is low). Otherwise, skewness is close to zero or slightly below zero in poor economic conditions. Thus, moving from the top to the medium value of the macro factor leads to a large skewness reduction but moving into the lowest tercile only leads to a marginal reduction in skewness. Panel (c) displays the conditional excess kurtosis.[21] Figure 7 presents the entire conditional distribution of target changes for the (*Low*, *Low*), (*Medium*, *Medium*) and (*High*, *High*) scenarios. As expected, the dispersion of the probabilities increases dramatically from high to low values of the macro factor. Looking at the entire distribution makes clear that the higher skewness associated with high values of the macro factor is not a reflection of higher uncertainty but, instead, a consequence of the trivial probability attributed to negative changes. In fact, the probability that the FOMC will not change its target is close to 0.85 when the target rate is high and economic conditions are good.

Cyclical uncertainty induces changes in the risk premium because target rate innovations are priced. The continuously compounded, expected excess return on the security with the payoff

[21]The Skellam distribution inherits from the Poisson distribution the property that its excess kurtosis is always strictly positive and inversely related to variance.

$\exp(-c'X_{t+1})$ is (see Le et al. (2003)),

$$E_t^{\mathbb{P}}\left[\log\frac{\exp(-c'X_{t+1})}{E_t[\exp(-r_t - c'X_{t+1})]}\right] = -c'Var_t^{\mathbb{P}}[X_{t+1}]\delta_t + o(c) + o(\lambda_t), \tag{30}$$

where c is the vector of risk exposures. This shows that changes in the compensation for risk can be caused by changes in the prices of risk, λ_t, or changes in the quantity of risk, $-c'Var_t^{\mathbb{P}}[X_{t+1}]$. The latter channel only affects compensation for target rate risk since the covariance matrix of the remaining state variables is constant (i.e. only the first line of $Var_t^{\mathbb{P}}[X_{t+1}]$ is not constant). The price of target rate risk, $\delta_{0,r}$, is negative and significant. Unexpectedly high target rates are associated with higher marginal utility states. This may reflect the oft-cited fear that the Fed will increase the target rate too much, or that it will not decrease it sufficiently, and depress economic conditions. In the model, this channel shifts the conditional dynamics of r_t between the $\mathbb{P}$ and $\mathbb{Q}$ measures through the adjustment factors $\exp(\delta_{0,r}\Delta) \approx 0.95$ and $\exp(-\delta_{0,r}\Delta) \approx 1.05$ (see Equation 21). This risk-adjustment pulls the components of λ_u closer to zero and pushes the components of λ_d away from zero. Empirically, this channel mainly works through an increase in the response to the macro factor under the risk-neutral measure. Expected target rates increase more when economic conditions improve and decrease more when economic conditions deteriorate.[22] Finally, cyclical uncertainty also induces changes in the spread between $\mathbb{P}$-variance and $\mathbb{Q}$-variance, the variance spread, when target rate innovations are priced. The conditional variance of target rates is more counter-cyclical (with respect to z_t) under the $\mathbb{Q}$ measure. On the other hand, the relationship with the target rate changes sign and the conditional $\mathbb{Q}$-variance is positively correlated with the target rate.

V The Liquidity Factor

A *The Liquidity Factor and Forward-to-Futures Spreads*

The liquidity factor captures common variations in forward-to-futures spreads across maturities. Table VII displays the results from regressions of the level and slope components extracted from spreads in Section II. The liquidity factor captures 36% of the daily variations in the slope component and 3% of the variations in the level component. Therefore, l_t explains at least 39% of the total daily variations of forward-to-futures spreads across all 6 maturities. The results are reversed for the macro factor. It explains 4% of the slope variations and 29% of the level variations.

[22] Note that the total risk premium associated with target rate exposure combine the effect of changes in $Var_t^{\mathbb{P}}[r_{t+1}] \times \delta 0, r$ and $Cov_t^{\mathbb{P}}[r_{t+1}, z_{t+1}] \times \lambda_{z,t}$.

Table VII: **Forward-to-Futures Spreads and State Variables**

Results from the regressions of the first two principal components (i.e. level and slope) extracted from forward-to-futures spreads. Regressors include different combination of state variables. Regressors are standardized with zero mean and unit variance. Newey-West (30 lags) standard errors in parentheses. Daily data from February 1994 to July 2007, but excluding the first 9 months of 1999 (i.e. the Millenium effect).

	r_t	s_t	z_t	l_t	R^2
				0.22	0.03
Level				(0.11)	
			0.69		0.29
			(0.08)		
	-0.87	0.00	1.54	0.70	0.56
	(0.098)	(0.024)	(0.106)	(0.075)	
				0.37	0.36
Slope				(0.03)	
			-0.12		0.04
			(0.045)		
	-0.17	0.02	0.12	0.41	0.39
	(0.060)	(0.012)	(0.071)	(0.039)	

Regressions on all state variables yield R^2s of 56% and 39% for the level and slope component, respectively. Importantly, other state variables do not add to the liquidity factor's explanatory power for the slope component. Since the slope component, but not the level component, predicts excess returns (see section II), this suggests that l_t plays a key role to predict the risk premium and improve Target rate forecasts.

B Sources of the Liquidity Factor Information Content

The liquidity factor provides a significant contribution to the improvement of target rate forecasts. To see this, take expectations in Equation 29 and re-arrange to obtain,

$$E_t\left[\bar{r}_n\right] = F(t,n) - E_t\left[xr_{t,n}\right], \tag{31}$$

which shows that improvements in forecasting performance, relative to naive futures-based forecasts, must come from the ability to predict excess returns. A similar relation holds for excess returns on forward LIBOR rates.[23] Note that the information content of the liquidity factor can originate in the futures market or in the LIBOR market. Transaction costs or limits to arbitrage in the futures

[23]More generally, it is useful to think of Equation 31 as a decomposition of the model's fit into (i) its ability to price futures rates accurately and (ii) its ability to model the passage from the risk-neutral to the historical dynamics (i.e. the risk premium). The first step relies on the no-arbitrage restrictions and the pricing equations to pin down parameters of the risk-neutral dynamics from the cross-section of rates. The second step estimates the historical of state variables and pins down risk parameters. The passage between the historical dynamics is then characterized by Equations 20 and 21.

market may induce price pressures that reveal changes in target rate expectations (Piazzesi and Swanson (2006)). These price deviations are mechanically reflected in excess returns. To see this, suppose that in reaction to economic news the observed futures rate deviates from the no-arbitrage rate in a frictionless economy, $\tilde{F}(t,n) = F(t,n) + \zeta_t(n)$. In this case, Equation 29 becomes

$$xr_{t,n} = F(t,n) + \zeta_t(n) - \bar{r}_n = F(t,n) - E_t[\bar{r}_n] - \epsilon_{t,n} + \zeta_t(n), \tag{32}$$

where $\zeta_t(n)$ is predictable. This term is specific to futures rates and affects forward-to-futures spreads. On the other hand, the effect of liquidity or credit conditions on LIBOR rates induces predictable excess returns in forward rates to compensate for liquidity or default risk. This leads to an equation similar to Equation 32 but for forward rates. Higher risks cause increases of forward-to-futures spreads. It may also predict lower policy rates through the anticipated reaction of the Fed to funding conditions in the LIBOR market.

Inspection of the liquidity factor time series hints at the relative importance of each effect. Figure 5(c) compares the liquidity and the macro factors. The liquidity factor exhibited peaks when financial markets were in turmoil: the Mexican Peso crisis (1994), the Asian crisis (1997), the failure of LTCM (1998) and fears of the Millennium bug. A natural interpretation is that, in these periods, forward-to-futures spreads reflect higher default risk or tense funding conditions on the interbank lending market. But these episodes represent only a small fraction of the (daily) sample. Still, Figure 5(c) shows that, perhaps surprisingly, the liquidity factor exhibits sharp but opposite co-movements with the macro factor *throughout* the sample. The sample correlation is -0.25 and parameter estimates also imply a negative correlation. Thus, improvements (deteriorations) in anticipated economic conditions are often associated with declines (increases) of forward-to-futures spreads but with no observed changes of conditions on the interbank market. In these cases, the futures market leads the LIBOR market. Futures rates increase (decrease) faster than LIBOR rates when broad economic conditions are improving (deteriorating). This suggests that the liquidity factor predicts lower policy rates and lower returns on futures because forward-looking information is revealed by participants in the futures market. There are several reasons why futures and LIBOR markets may not be fully integrated. LIBOR loans are fully funded while futures contracts require no exchange of principal. Also, LIBOR loans are uncollateralized while futures contracts are cleared via a central counter-party where variation in margins minimizes the default exposures of each participant. Finally, participation in the LIBOR market is limited and LIBOR positions cannot be reversed as easily, especially in periods of turmoil.

C Tests of Alternative Sources of Information Content

This section tests the two alternative interpretations of the liquidity factor introduced above. Both channels predict a positive correlation between the liquidity factor and future target rates but they *differ* in their implications for excess returns. Hedging demand in the futures market induces predictable *negative* excess returns on that market *only*. In contrast, tensions in the inter-bank market induce *positive* predictable excess returns in LIBOR rates *only*. I test the first hypothesis via predictive regressions for futures excess returns,

$$xr^{Fut}_{t,n} = \gamma_{0,n} + \gamma_{l,n} l_t + u_{t,n}, \tag{33}$$

where l_t is the liquidity factor and we should have $\gamma_{l,n} < 0$. I test the second hypothesis via similar predictive regressions but for forward rates returns,

$$xr^{For}_{t,n} = \gamma_{0,n} + \gamma_{l,n} l_t + u_{t,n}. \tag{34}$$

where we should have $\gamma_{l,n} > 0$. As in Section II, I compute excess returns at a monthly frequency to avoid econometric issues related to overlapping observations and long-horizon predictability regressions. Table VIII displays the results.

Monthly excess returns on futures averaged between 2 and 4 bps (annualized) across maturites. These are consistent with results in Hamilton (2009), though slightly lower. The information content of liquidity is significant and large. Also, coefficients have the expected sign. A one-standard deviation increase in the liquidity factor decreases excess returns between 3 and 6 bps for maturities beyond 2 months and the R^2 ranges from 5% to 7%. Results from forward excess returns show no evidence of predictability from the liquidity factor and coefficients typically have the wrong sign. The evidence points toward transitory hedging pressures on the futures market as the main driving force behind the liquidity factor.

D The Role of Hedging Demand

Piazzesi and Swanson (2006) show that futures excess returns can be predicted using net long positions of non-commercial participants on the futures market. They argue that "hedgers -primarily banks- essentially paid an insurance premium to non-commercial participants for providing hedging services". They suggest that non-commercial participants face limits on the size of the positions they may take. Alternatively, non-commercial participants may be risk averse and the hedging activity of banks may be associated with high marginal utility states. An important question, then, is whether the information content of the liquidity factor is shared with variations in the net long

Table VIII: **Predictive Regressions : Forward vs Futures Excess Returns**

Results from predictive regressions of monthly futures and forward excess returns on the liquidity factor, l_t. Excess returns are in basis points (annualized). Coefficient estimates provide the change in expected excess returns due to a change of one standard deviation in l_t. Newey-West t-statistics (6 lags) in parenthesis and R^2 in brackets.

(a) $xr^{Fut}_{t,n} = \gamma_{0,n} + \gamma_{l,n} l_t + u_{t,n}$

	2	**3**	**4**	**5**	**6**
γ_0	2.20	2.16	2.75	3.02	3.52
	(2.27)	(1.71)	(1.80)	(1.69)	(1.77)
γ_l	-1.38	-3.22	-3.98	-4.94	-5.93
	(-1.03)	(-1.93)	(-1.93)	(-2.16)	(-2.42)
R^2	[1.4]	[5.3]	[5.2]	[5.9]	[6.5]

(b) $xr^{For}_{t,n} = \gamma_{0,n} + \gamma_{l,n} l_t + u_{t,n}$

	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
γ_0	9.97	7.65	5.71	8.15	3.90	11.03	6.91	6.12	15.71	8.14	7.41
	(6.26)	(5.08)	(2.50)	(3.80)	(1.61)	(3.42)	(2.50)	(1.84)	(4.36)	(2.04)	(1.92)
γ_l	-2.80	-1.25	0.05	-0.90	1.92	-1.66	-1.72	1.87	-1.24	-0.65	1.59
	(-2.15)	(-1.03)	(0.03)	(-0.53)	(0.81)	(-0.58)	(-0.72)	(0.69)	(-0.43)	(-0.20)	(0.51)
R^2	[2.3]	[0.4]	[0.0]	[0.1]	[0.6]	[0.3]	[0.4]	[0.3]	[0.1]	[0.0]	[0.1]

positions of non-commerical hedgers [NLP_t].[24]

I report results from predictive regressions on different combinations of the liquidity factor, l_t, the macro factor, z_t, and NLP_t. Table IX(a) presents results based on NLP_t only. Its predictive content is similar to that of l_t with R^2 ranging between 2% and 6%. Consistent with Piazzesi and Swanson (2006), NLP_t is positively related with excess returns. A one-standard deviation increase predicts increases in returns of between 2 and 6 bps across maturities. Panel IX(b) presents results from regressions combining l_t and NLP_t. The R^2s are higher ranging between 7% and 9% between maturities of 3 and 6 months but less than the sum of the univariate R^2's. Similarly, coefficient estimates are lower relative to univariate regressions. This suggests that these predictors overlap substantially. Panel IX(c) presents results from regressions combining l_t and z_t and shows no improvement in preditability relative to using l_t only. Finally, Panel IX(d) presents results from regressions combining l_t and z_t and NLP_t. The macro factor becomes marginally significant and net long positions of non-commercial investors is significant at horizons between 2 and 6 months. The results contrast with standard results showing no predictability for the month-ahead contract.

[24] I follow Piazzesi and Swanson (2006) and use eurodollar futures position data. The position data are published weekly with a three-day lag by the U.S. CFTC. I match the daily data with the weekly CFTC mandatory reporting date to construct a weekly sample of factors, returns and positions.

Table IX: **Predictive Regressions : The Role of Hedging Demand**

Results from predictive regressions of monthly futures excess returns on different combinations of liquidity factor, l_t, the macro factor, z_t and the net long position by non-commercial investors, NLP_t. Excess returns are in basis points (annualized). Coefficient estimates provide the change in expected excess returns due to a change of one standard deviation in the regressors. Newey-West t-statistics (6 lags) in parenthesis and R^2 in brackets.

(a) $xr^{Fut}_{t,n} = \gamma_{0,n} + \gamma_{nlp} NLP_t + u_{t,n}$

	2	**3**	**4**	**5**	**6**
γ_0	2.20	2.16	2.75	3.02	3.52
	(2.20)	(1.59)	(1.70)	(1.61)	(1.71)
γ_{nlp}	1.60	2.84	3.88	4.66	5.64
	(1.17)	(1.61)	(1.96)	(2.13)	(2.43)
R^2	[1.9]	[4.1]	[5.0]	[5.3]	[5.9]

(b) $xr^{Fut}_{t,n} = \gamma_{0,n} + +\gamma_l l_t + \gamma_{nlp} NLP_t u_{t,n}$

	2	**3**	**4**	**5**	**6**
γ_0	2.20	2.16	2.75	3.02	3.52
	(2.30)	(1.76)	(1.88)	(1.79)	(1.92)
γ_l	-0.90	-2.50	-2.92	-3.69	-4.41
	(-0.81)	(-1.80)	(-1.57)	(-1.77)	(-1.95)
γ_{nlp}	1.26	1.88	2.75	3.24	3.94
	(1.10)	(1.35)	(1.66)	(1.74)	(1.94)
R^2	[2.4]	[6.8]	[7.4]	[8.1]	[9.0]

(c) $xr^{Fut}_{t,n} = \gamma_{0,n} + \gamma_z z_t + \gamma_{nlp} l_t + u_{t,n}$

	2	**3**	**4**	**5**	**6**
γ_0	2.20	2.16	2.75	3.02	3.52
	(2.28)	(1.71)	(1.78)	(1.68)	(1.75)
γ_z	0.67	0.78	0.80	1.02	1.01
	(0.71)	(0.66)	(0.50)	(0.54)	(0.49)
γ_l	-1.24	-3.06	-3.82	-4.73	-5.72
	(-0.84)	(-1.68)	(-1.70)	(-1.91)	(-2.20)
R^2	[1.7]	[5.6]	[5.5]	[6.2]	[6.7]

(d) $xr^{Fut}_{t,n} = \gamma_{0,n} + \gamma_z z_t + \gamma_l l_t + \gamma_{nlp} NLP_t + u_{t,n}$

	2	**3**	**4**	**5**	**6**
γ_0	2.20	2.16	2.75	3.02	3.52
	(2.40)	(1.82)	(1.93)	(1.84)	(1.96)
γ_z	2.30	3.07	3.93	4.77	5.41
	(1.83)	(1.86)	(1.72)	(1.83)	(1.97)
γ_l	0.16	-1.08	-1.11	-1.50	-1.92
	(0.12)	(-0.63)	(-0.47)	(-0.57)	(-0.73)
γ_{nlp}	2.77	3.91	5.35	6.38	7.51
	(2.07)	(2.18)	(2.40)	(2.52)	(2.86)
R^2	[4.7]	[9.7]	[10.4]	[11.3]	[12.2]

Strikingly, combining macro conditions and hedging demand captures the information content of the liquidity factor. This is significant. Conditional on macro conditions, hedging demand on the futures market captures the information content of the liquidity factor.

For comparison with existing results, Table IX report results based on hold-to-maturity excess returns. The results are qualitatively similar but quantitatively stronger with R^2s ranging between 5% and 19% when using l_t and between 4% and 11% when using NLP_t across maturities between 3 and 6 months. Estimated coefficients are also substantially higher. Again, hedging demand and the macro factor, *together*, capture the information content of the liquidity factor. Their combined predictive power is much higher than documented elsewhere, ranging between 4% at a maturity of 2 months to 35% at a maturity of 6 months. Interestingly, there is even evidence of predictability for the current-month contract. Again, the evidence shows that hedging demand plays a predominant role behind the information content of futures rates. Note that this does not preclude that variations in default risk implicit in LIBOR rates cannot affect the liquidity factor. What the results show is that variations of forward-to-futures spreads which are informative for future target rates originated from the futures market *in this sample*. Clearly, shocks to LIBOR rates likely played an important role in the events of 2007-2009.

VI Conclusion

This paper provides a dynamic term structure model adapted to the empirical features of policy rates in many countries. The support of target rate changes is discrete and changes occur infrequently, almost always following scheduled policy meetings. Estimation from U.S. dollar LIBOR rates and fed funds futures rates in a daily sample from 1994 to 2007 produces several novel results. The model provides unbiased target rate forecasts up to one year ahead. The accuracy of forecasts is at par with benchmark predictive regressions for horizons up to 6 months and improves at longer horizons. This implies that the model provides an accurate decomposition of short-term interest rates into an expectation and a risk premium component. Moreover, the model uncovers significant cyclical changes in the uncertainty surrounding the policy response to economic conditions. I argue that transitory deviations between LIBOR and futures rates which are informative for the evolution of target rates reflect demand pressure on the intermediation mechanism from participants seeking to hedge exposures to future interest rates. The results show that the liquidity premium in interest rates is relevant to macroeconomic and finance researchers alike. Also, the novel specification introduced above can be extended in several directions. The model could be adapted to address the extreme events in the LIBOR market during the 2007-2009 crisis. Also, it may be possible to extend the forecasting performance toward longer horizons using Eurodollar futures at longer

Table X: **Predictive Regressions: Excess Holding Returns**

Results from predictive regressions of futures excess holding returns. Predictors include the macro factor, z_t, the liquidity factor, l_t, and the net position of non-commercial participants, NLP_t. Regressors are centered around zero and normalized by their standard deviations. Excess returns are in basis points (annualized). Coefficient estimates provide the change in expected excess returns due to a change of one standard deviation in the regressors. I include t-statistics based on Newey-West standard errors (6 lags) in parenthesis and R^2 in brackets.

(a) $xr_{t,n} = \gamma_{0,n} + \gamma_{l,n} l_t + u_{t,n}$

	1	**2**	**3**	**4**	**5**	**6**
$\gamma_{0,n}$	0.78	2.91	5.25	8.52	12.32	16.40
	(2.28)	(3.58)	(3.69)	(4.00)	(4.36)	(4.55)
$\gamma_{l,n}$	-0.11	-1.39	-4.25	-8.08	-14.38	-20.33
	(-0.34)	(-1.45)	(-2.58)	(-3.30)	(-4.55)	(-5.12)
R^2	[0.0]	[1.5]	[5.3]	[9.1]	[15.9]	[19.0]

(b) $xr_{t,n} = \gamma_{0,n} + \gamma_{nlp,n} NLP_t + u_{t,n}$

	1	**2**	**3**	**4**	**5**	**6**
$\gamma_{0,n}$	0.78	2.91	5.25	8.52	12.32	16.40
	(2.27)	(3.57)	(3.62)	(3.89)	(4.12)	(4.23)
$\gamma_{nlp,n}$	-0.07	1.33	3.88	7.22	11.06	15.55
	(-0.18)	(1.18)	(1.91)	(2.48)	(2.93)	(3.38)
R^2	[0.0]	[1.4]	[4.4]	[7.3]	[9.4]	[11.1]

(c) $xr_{t,n} = \gamma_{0,n} + \gamma_{z,n} z_t + \gamma_{l,n} l_t + \gamma_{nlp,n} NLP_t + u_{t,n}$

	1	**2**	**3**	**4**	**5**	**6**
$\gamma_{0,n}$	0.78	2.91	5.25	8.52	12.32	16.40
	(2.33)	(3.66)	(3.83)	(4.24)	(4.71)	(5.04)
$\gamma_{z,n}$	1.08	2.38	5.77	10.12	15.38	22.09
	(2.00)	(2.13)	(3.06)	(3.40)	(3.86)	(4.28)
$\gamma_{l,n}$	0.39	0.17	-0.32	-1.12	-4.10	-5.65
	(0.93)	(0.18)	(-0.21)	(-0.45)	(-1.23)	(-1.25)
$\gamma_{nlp,n}$	0.60	2.54	6.55	11.69	16.96	24.11
	(1.44)	(2.10)	(2.83)	(3.36)	(3.89)	(4.44)
R^2	[1.2]	[4.5]	[12.5]	[19.7]	[28.7]	[34.7]

maturities. Finally, the model can be brought closer to macro-finance model of the term structure if we introduce stronger structural assumptions.

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Appendix

A Estimation

A.1 Data

The Fed targets the overnight fed funds rate. The effective rate is published by the Board of Governors of the Federal Reserve System (see statistical release H.15). It is the weighted average of rates on brokered unsecured fed funds overnight loans between large banks. The payoff of a futures contract depends on the effective rates in a given reference period. Quoted LIBOR rates are annualized (actual/360) but not compounded. I compute the actual daycount to maturity using the BBA / ISDA Modified Business Days convention. Futures rates are computed from quoted prices as $F(t,n) = (100 - P(t,n))/3600$ following the CME's convention. Each rate is converted to a continuously compounded daily rate. I use the CME "Following Business Days" convention to determine the monthly settlement date.

A.2 State-Space Representation

The model can be written as a state-space system for the purpose of estimation. The transition equation is given by the VAR representation in Equation 13. The measurement equation includes LIBOR rates and futures rates, stacked in vector Y_t, and is given by

$$Y_t = \Upsilon(t, X_t) + \epsilon_t, \tag{35}$$

where $\Upsilon(\cdot,\cdot)$ is the model prediction for rates and $\epsilon_{i,t}$ are i.i.d. mean zero Gaussian pricing errors with standard deviation ω_i. The measurement equation also includes the target rate and the effective spread, stacked together with yields in the vector $\tilde{Y_t} = [r_t \; s_t \; Y_t]^T$. The joint conditional likelihood of $\tilde{Y_t}$ is available in closed form (see appendix A.4) but two of the state variables are unobserved. The Kalman filter is not applicable since futures rates are non-linear functions of the states and, moreover, the optimal filter is not available in closed-form. Instead, I combine QML estimation with the Unscented Kalman Filter [UKF].

A.3 Unscented Kalman Filter

The UKF provides an approximation to non-linear transformations of a probability distribution. It has a Monte Carlo flavour but the sample is drawn according to a deterministic algorithm. It reduces the computational burden considerably, relative to simulation-based methods, but provides greater accuracy than linearization (Christoffersen et al. (2007)). The UKF has been introduced in Julier et al. (1995) and Julier and Uhlmann (1996) (see Wan and der Merwe (2001) for textbook treatment) and was first imported in finance by Leippold and Wu (2003).

Given, $\hat{X}_{t+1|t}$, a time-t forecast of X_{t+1}, and its associated MSE, $\hat{Q}_{t+1|t}$, the filter selects a set of sigma points in the distribution of $X_{t+1|t}$ such that

$$\bar{\mathbf{x}} = \sum_i w^{(i)} x^{(i)} = \hat{X}_{t+1|t} \quad \text{and} \quad \mathbf{Q}_x = \sum_i w^{(i)} (x^{(i)} - \bar{\mathbf{x}})(x^{(i)} - \bar{\mathbf{x}})' = \hat{Q}_{t+1|t}.$$

Julier et al. (1995) proposed the following set of sigma points and weights,

$$x^{(i)} = \begin{cases} \bar{\mathbf{x}} & i = 0 \\ \bar{\mathbf{x}} + \left(\sqrt{\frac{N_x}{1-w^{(0)}} \sum_x}\right)_{(i)} & i = 1, \ldots, K \\ \bar{\mathbf{x}} - \left(\sqrt{\frac{N_x}{1-w^{(0)}} \sum_x}\right)_{(i-K)} & i = K+1, \ldots, 2K \end{cases} \qquad w^{(i)} = \begin{cases} w^{(0)} & i = 0 \\ \frac{1-w^{(0)}}{2K} & i = 1, \ldots, K \\ \frac{1-w^{(0)}}{2K} & i = K+1, \ldots, 2K \end{cases}$$

where $\left(\sqrt{\frac{N_x}{1-w^{(0)}}\sum_x}\right)_{(i)}$ is the i-th row or column of the matrix square root. Julier and Uhlmann (1996) use a Taylor expansion to evaluate the approximation's accuracy. The expansion of $y = g(x)$ around $\bar{x}$ is

$$\bar{y} = E\left[g(\bar{x} + \Delta x)\right] = g(\bar{x}) + E\left[D_{\Delta x}(g) + \frac{D^2_{\Delta x}(g)}{2!} + \frac{D^3_{\Delta x}(g)}{3!} + \cdots\right],$$

where the $D^i_{\Delta x}(g)$ operator evaluates the total differential of $g(\cdot)$ when perturbed by Δx, and evaluated at $\bar{x}$. A useful representation of this operator in our context is

$$\frac{D^i_{\Delta x}(g)}{i!} = \frac{1}{i!}\left(\sum_{j=1}^{n} \Delta x_j \frac{\partial}{\partial x_j}\right)^i g(x)\Bigg|_{x=\bar{x}}.$$

Different approximation strategies for $\bar{y}$ will differ by either the number of terms used in the expansion or the set of perturbations Δx. If the distribution of Δx is symmetric, all odd-ordered terms are zero and we can re-write the second terms as a function of the covariance matrix P_{xx} of Δx,

$$\bar{y} = g(\bar{x}) + \left(\nabla^T P_{xx} \nabla\right) g(\bar{x}) + E\left[\frac{D^4_{\Delta x}(g)}{4!} + \cdots\right].$$

Linearisation leads to the approximation $\hat{\bar{y}}_{lin} = g(\bar{x})$ while the unscented approximation is exact up to the third-order term. In the Gaussian case, Julier and Uhlmann (1996) show that same-variable fourth moments agree as well and that all other moments are lower than the true moments of Δx. Then, approximation errors of higher order terms are necessarily smaller for the UKF than for the EKF. Using a similar argument, Julier and Uhlmann (1996) show that linearization and the unscented transformation agree with the Taylor expansion up to the second-order term and that approximation errors in higher-order terms are smaller for the UKF.

A.4 Quasi-Maximum Likelihood Estimation

A Quasi-Maximum Likelihood [QML] estimator is feasible. The joint log-likelihood is given by

$$\begin{aligned} L(\Theta; Y) &= \sum_{t=1}^{T} \log\left(f_t\left(\tilde{Y}_t | \tilde{Y}_{t-1}; \theta\right)\right) \\ &= \sum_{t=1}^{T} \log\left(f_t(Y_t | \hat{z}_{t|t-1}, \hat{l}_{t|t-1}, r_t, s_t) f(r_t | r_{t-1}, \hat{z}_{t|t-1}, \hat{l}_{t|t-1}, I_t) f(s_t | s_{t-1})\right) \end{aligned}$$

where all model parameters are grouped in the vector Θ. The density of Y_t depends on time through the deterministic schedule of FOMC meetings. Note the separation of the marginal likelihood of s_t implies that its dynamics can be estimated separately. Results do not change materially if we proceed via joint estimation. On the other hand, separate estimation lightens the computation burden, improves numerical optimization and leads to a more accurate Hessian matrix because of the weak link between s_t and yields in the data. The following constraints were imposed on the parameter space. First, I set $\mu_z = 0$ to identify the level of z_{t+1} separately from that of $\lambda_{z,u}$ and $\lambda_{z,d}$. Second, I impose $\lambda_{z,u} > 0$ and $\lambda_{z,d} > 0$ to identify the sign of z_{t+1}. The level and sign of l_{t+1} are pinned down by the observed forward-to-futures spreads. Third, the eigenvalues of the matrix $\Phi(I_{t+1}) = 1$ must lie within the unit circle. We have that $\phi_{s,r} = \phi_{z,r} = \phi_{l,r} = 0$ then the policy function induces reversion to the mean $\lambda_{u,r} < 0$ and $\lambda_{d,r} < 0$ so that $\phi_r(1) < 1$. Joint mean-reversion follows if s_t, z_t and l_t are jointly stationary. I impose that $|\phi_s| < 1$. In practice I estimate the model based on a re-parameterized likelihood where $\tilde{\phi} = [I - \phi]^{-1}$

$$\tilde{\phi} = [I - \phi]^{-1} = \left[I - \begin{bmatrix} \phi_z & \phi_{z,l} \\ \phi_{l,z} & \phi_l \end{bmatrix}\right]^{-1}, \tag{36}$$

and the only remaining restriction is that $\tilde{\phi}$ must be invertible. Also, the jump component of the effective rate is well-defined only if $\lambda_{J,s} \geq 0$. I impose that $\lambda_{z,d} \geq 0$ and $\lambda_{z,u} \geq 0$ to identify the sign of z_t. More importantly, λ_t^u and λ_t^d must remain non-negative so that the distribution of target jumps remains well defined. These constraints cannot be easily imposed on the parameter space as they can only be checked recursively as we filter the state variables. In practice I impose that $\hat{\lambda}_t^i = \max\left(0, \lambda_t^i\right)$. This leaves state variables unrestricted but constrains the policy function. The restriction is reasonable. As λ_t^i approaches zero, the probability distribution of the corresponding jump n_t^i approaches the trivial distribution with a unit mass at zero. When it reaches zero, the policy function becomes one-sided and can then be summarized as a Poisson distribution.

The constrained QML estimator is given by

$$\hat{\Theta}^{QML} = \underset{\Theta}{\text{argmax}}\ L(\theta; Y) \quad s.t. \quad \Theta \in S$$

where $S \subseteq \mathbb{R}^K$and we have $\hat{\Theta} \sim N(\Theta_0, T^{-1}\Omega)$ for some true parameter value, Θ_0, in the interior of the parameter space. Optimization of the log-likelihood is carried out using the active-set algorithm from the IMSL Fortran optimization library. I report estimates for $\hat{\phi} = I - \hat{\tilde{\Phi}}^{-1}$ and the p-values associated with Wald statistics computed for the nulls that each individual coefficient is zero. Finally, as in Piazzesi (2005a), λ_0^u and λ_0^d are poorly estimated because of the lack of policy changes outside of scheduled FOMC meetings. I calibrate them so that the distribution of policy moves is symmetric outside of scheduled FOMC meetings and that the variance of target changes matches the sample variance. Results are robust to the choice of calibration strategy.

B Conditional Laplace Transform

B.1 Skellam Distribution

The dynamics of state variables is summarized by Equations 3, 11 and 12. Although the process for r_t is novel, its Laplace transform and its density are known in closed-form. Conditional on the state, changes in the target rate follow a Skellam distribution (Skellam (1946), Johnson et al. (1997)). This distribution is characterized as the difference between two independent Poisson random variables. Consider two univariate Poisson variables N_1 and N_2 with parameters λ_1 and λ_2, respectively. The Laplace transform of their difference $Z \equiv N_1 - N_2$ is

$$T(u, Z) = \exp\left(\lambda_1(e^u - 1) + \lambda_2(e^{-u} - 1)\right), \quad u \in \mathbb{R},$$

while its probability mass function is

$$f(Z = z) = \exp(-(\lambda_1 + \lambda_2)) \left(\frac{\lambda_1}{\lambda_2}\right)^{z/2} I_z\left(2\sqrt{\lambda_1 \lambda_2}\right),$$

where $I_k(y)$ is the modified Bessel function of the first kind. In our context, the coefficients of the conditional Laplace transform vary through time because of the evolution of the underlying state variables and because of the (deterministic) variation in the FOMC meeting schedule. Hence, computation of the transform depends on the occurrence of a FOMC meeting in the next period and the two cases must be treated separately.

B.2 No FOMC Meeting

The joint conditional transform when no meeting is scheduled to occur is given by,

$$T(u, X_{t+1}|I_{t+1} = 0) \equiv E_t\left[\exp\left(u^T X_{t+1}\right)|I_{t+1} = 0\right] = \exp\left(A(I_{t+1} = 0, u) + B(I_{t+1} = 0, u)^T X_t\right),$$

where I_t is equal to 1 if a meeting occurs at time-t and 0 otherwise. Coefficients are given by

$$\begin{aligned}
A(I_t = 0, u) &= g_0(u_r) + u^T \mu(I_{t+1} = 0) + \frac{1}{2} u^T \Omega u + \lambda_s \left(T(u_s, Y^s_{t+1}) - 1 \right) \\
B(I_t = 0, z) &= \Phi(I_{t+1} = 0) u,
\end{aligned}$$

where $\mu(I_{t+1} = 0)$ and $\Phi(I_{t+1} = 0)$ are defined in the text, $g_0(x) \equiv \lambda^u_0(e^{\Delta x} - 1) + \lambda^d_0(e^{-\Delta x} - 1)$, and

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_s^2 & \sigma_{s,z} & \sigma_{s,l} \\ 0 & \sigma_{s,z} & \sigma_z^2 & \sigma_{z,l} \\ 0 & \sigma_{s,l} & \sigma_{z,l} & \sigma_l^2 \end{bmatrix}.$$

B.3 FOMC Meeting

The joint conditional Laplace transform in the case when a FOMC meeting is scheduled to occur, and conditional on the realization of s_{t+1}, z_{t+1} and l_{t+1}, is given by,

$$\begin{aligned}
T(u, X_{t+1} | I_{t+1} = 1) \equiv& E_t \left[\exp\left(u^T X_{t+1} \right) | I_{t+1} = 1 \right] = E_t \left[E_{s_{t+1}, z_{t+1}, l_{t+1}} \left[\exp\left(u^T X_{t+1} \right) | I_{t+1} = 1 \right] \right] \\
=& E_t \left[\exp\left(-G(u_r)^T \bar{X} + \lambda(e^{\Delta u_r} + e^{-\Delta u_r} - 2) + G(u_r)^T X^*_{t+1} + u^T X^*_{t+1} \right) \right],
\end{aligned}$$

where $X^*_{t+1} = [r_t \ s_{t+1} \ z_{t+1} \ l_{t+1}]$, the function $G(x)$ is given by

$$G(x) = [g_r(x) \ g_r(x) \ g_z(x) \ g_l(x)]^T, \ x \in \mathbb{R},$$

and the functions $g_k(y), k = r, s, z$, are defined as

$$g_k(x) \equiv \left(\lambda_{u,k}(e^{x\Delta} - 1) - \lambda_{d,k}(e^{-x\Delta} - 1) \right).$$

We then have that

$$T(u, X_{t+1} | I_{t+1} = 1) = \exp\left(A(I_t = 1, u) + B(I_t = 1, u)^T X_t \right),$$

with coefficients

$$\begin{aligned}
A(1, u) =& - G(u_r)^T \bar{X} + h_1(u_r) + \left(G(u_r) + u \right)^T \mu(I_{t+1} = 1) + \frac{1}{2} \left(G(u_r) + u \right)^T \Omega \left(G(u_r) + u \right) \\
&+ \lambda_s \left(T(g_s(u_r) + u_s, Y^s_{t+1}) - 1 \right) \\
B(1, u) =& \Phi(I_{t+1} = 0) \left(G(u_r) + u \right),
\end{aligned}$$

where $h_1(x) \equiv \lambda(e^{\Delta x} - 1) + \lambda(e^{-\Delta x} - 1)$.

B.4 Risk-Neutral Distribution

The joint conditional Laplace transform under $\mathbb{Q}$ is given by,

$$\begin{aligned}
T^Q(u, X_{t+1}) &= E_t \left[M_{t,t+1} \exp\left(u^T X_{t+1} \right) \right] \\
&= \exp\left(A^Q(I_{t+1}, u) + (B^Q(I_{t+1}, u) - C_r)^T X_t \right),
\end{aligned}$$

where $C_r = [1\ 1\ 0\ 0]^T$ and where coefficients are given by

$$\begin{aligned}
A^Q(0,u) =& g_0(u_r+\delta_{0,r}) - g_0(\delta_{0,r}) + u^T(\mu(I_{t+1}=0)+\Omega\delta_0) + \frac{1}{2}u^T\Omega u \\
&+ \lambda_s\left(T(u_s+\delta_{0,s},Y^s_{t+1}) - T(\delta_{0,s},Y^s_{t+1})\right) \\
A^Q(1,u) =& g_1(u_r+\delta_{0,r}) - g_1(\delta_{0,r}) - \left(G(u_r+\delta_{0,r}) - G(\delta_{0,r})\right)^T \bar{X} \\
&- \frac{1}{2}G(\delta_{0,r})^T\Omega G(\delta_{0,r}) + \frac{1}{2}\left(G(u_r+\delta_{0,r})+u\right)^T\Omega\left(G(u_r+\delta_{0,r})+u\right) \\
&+ \left(G(u_r+\delta_{0,r})+u\right)^T\left(\mu(I_{t+1}=1)+\Omega\delta_0\right) \\
&+ \lambda_s\left(T(g_s(u_r+\delta_{0,r})+\delta_{0,s}+u_s,Y^s_{t+1}) - T(g_s(\delta_{0,r})+\delta_{0,s},Y^s_{t+1})\right)
\end{aligned}$$

and

$$\begin{aligned}
B^Q(0,u) &= \left(\Phi(I_{t+1}=0)+\Omega\delta_1\right)u \\
B^Q(1,u) &= \left(\Phi(I_{t+1}=1)+\Omega\delta_1\right)\left(G(u_r+\delta_{0,r}) - G(\delta_{0,r}) + u\right).
\end{aligned}$$

C Generating Function for Prices

Consider the price at time-t of the payoff $\exp(u^T X_{t+m})$ at maturity m,

$$\begin{aligned}
\Gamma(u,t,m) &= E_t\left[M_{t,t+m}\exp\left(u^T X_{t+m}\right)\right] \\
&= E_t\left[M_{t+1}\Gamma\left(u,t+1,m-1\right)\right].
\end{aligned}$$

Substituting the guess $\Gamma(u,t,m) = \exp\left(c_0(u,t,m) + c(u,t,m)^T X_t\right)$ gives

$$\begin{aligned}
\Gamma(u,t,m) =& E_t\left[M_{t+1}\exp\left(c_0(u,t+1,m-1) + c(u,t+1,m-1)^T X_{t+1}\right)\right] \\
=& \exp\left(c_0(u,t+1,m-1) + A^Q(I_{t+1},c(u,t+1,m-1))\right) \\
&\times \exp\left(\left[B^Q(I_{t+1},c(u,t+1,m-1)) - C_r\right]^T X_t\right),
\end{aligned}$$

which implies the following recursion that coefficients must solve:

$$\begin{aligned}
c_0(u,t,h) &= c_0(u,t+1,h-1) + A^Q(I_{t+1},c(u,t+1,h-1)) \\
c(u,t,h) &= B^Q(I_{t+1},c(u,t+1,h-1)) - C_r,
\end{aligned}$$

for $0 \leq h \leq m$. Note that $\Gamma(u,t,0) = \exp(u^T X_t)$ implies $c_0(u,t,0) = 0$ and $c(u,t,0) = u$ for arbitrary $t \geq 1$ and u.

D Computing Recursions

The recursions in Equation 19 have time and maturity dimensions because the FOMC meeting schedule changes over time. Future meetings get closer by one day every day. At first, it would seem that a different recursion must be computed to match each (t,m) pair, implying a dramatic increase in computing costs. Fortunately, there is a way around this. The key is to note that future meeting dates are known in advance and, therefore, that coefficients $A(\cdot)$ and $B(\cdot)$ are not stochastic. Then, for a given date, and for given parameter values, we can compute coefficients for the price of an asset maturing on this date but for past observation dates[25] since the recursions are increasing in m but decreasing in t. As an example, consider the price, at some date $t+h$, of an asset maturing on that day. Its price is $\exp(u^T X_{t+h})$, and the coefficients,

[25] Another approach that reduces computing cost is to assume a constant time interval between meetings beyond the nearest schedule meeting date, as in Piazzesi (2005a). However, the use of fed funds futures contracts makes this approximation problematic as it may place some future meetings in the wrong month, implying severe mispricing of the corresponding futures contracts.

$c_0(u, t+h, 0)$ and $c(u, t+h, 0)$ correspond to the initial values in the recursions. Next, consider the price of that asset on the previous day. Its maturity is now one and the coefficients, $c_0(u, t+h-1, 1)$ and $c(u, t+h-1, 1)$, are given directly from the recursions above. We can then work our way back until we reach t, the first date in the sample where an asset matures at time $t+h$. Finally, varying the maturity date, t, provides us with all the needed coefficients[26].

E Asset Prices

E.1 Discount Bonds

The case $u = 0$ corresponds to the price, $D(t, m)$, of a risk-free discount bond with maturity m,

$$D^{rf}(t, m) \equiv \Gamma(0, t, m) = E_t[M_{t,t+m}] = \exp\left(d_0^{rf}(t, m) + d^{rf}(t, m)^T X_t\right),$$

where $d_0^{rf}(t, m) \equiv c_0(0, t, m)$ and $d^{rf}(t, m) \equiv c(0, t, m)$.

E.2 LIBOR loan

A LIBOR loan is an asset with unit payoff which is further discounted at the rate l_t to offer compensation for illiquidity or counterparty risk. The price of a LIBOR loan can also be obtained from the price generating function by noting that

$$D^L(t, m) = E_t\left[M_{t,t+m}\exp\left(-\sum_{i=0}^{m-1} l_{t+i}\right)\right] = E_t\left[M^L_{t,t+m}\right],$$

with $M^L_{t+i} = M_{t+i}\exp(-l_{t+1})$. I guess and verify that the solution is exponential-affine, $D^L(t, m) = \exp\left(d_0^L(t, m) + d^L(t, m)^T X_t\right)$ with solution

$$\begin{aligned}
d_0^L(t, m) &= d_0^L(t+1, m-1) + A^Q\left(I_{t+1}, d^L(t+1, m-1)\right) \\
d^L(t, m) &= B^Q\left(I_{t+1}, d^L(t+1, m-1)\right) - C_L,
\end{aligned}$$

where $C_L = [1\ 1\ 0\ 1]^T$. Finally, note that $D^L(t, 0) = 1$ implies that $d_0^L(t, 0) = 0$ and $d^L(t, 0) = 0$ for any $t \geq 1$.

E.3 Singleton Futures Price

A difficulty arises when computing a singleton futures rate because the reference date $t+m$, at which the payoff is determined, is not generally the same than the settlement date $t+T$, at which the payment is made. That is, we have

$$\begin{aligned}
f(t, m, T) &= E_t[M_{t,t+T} r_{t+m}] = \\
&= \frac{\partial}{\partial u} E_t\left[M_{t,T}\exp(u r_{t+m})\right]\Big|_{u=0} \equiv \frac{\partial}{\partial u}\Gamma_f(u, t, m, T)\Big|_{u=0},
\end{aligned}$$

where $m \leq T$ and $u \in \mathbb{R}$. However, we can use the law of iterated expectations to obtain,

$$\begin{aligned}
\Gamma_f(u, t, m, T) &= E_t\left[M_{t,t+m}\exp(u r_{t+m}) E_{t+m}\left[M_{t+m,t+T}\right]\right] \\
&= E_t\left[M_{t,t+m}\exp(u r_{t+m}) D(t+m, T-m)\right] \\
&= \exp(d_0^{rf}(t+m, T-m))\Gamma(d^{rf}(t+m, T-m) + uC_r, t, m).
\end{aligned}$$

[26]This implies that some recursions must be started for some date $t+h$ beyond the end of the sample. Coefficients are discarded as we proceed backward in time until we reach the last observation date of the sample.

We can then use the results above to obtain

$$\Gamma_f(u,t,m,T) = \exp\left(d_0^{rf}(t+m,T-m)\right)$$
$$\times \exp\left(c_0(d^{rf}(t+m,T-m)+uC_r,t,m)+c(d^{rf}(t+m,T-m)+uC_r,t,m)^T X_t\right).$$

Taking the partial derivatives with respect to u and evaluating at $u=0$, the singleton futures rate is

$$f(t,m,T) = \exp\left(d_0^{rf}(t+m,T-m)+c_0\left(u^*,t,m\right)+c\left(u^*,t,m\right)^T X_t\right)$$
$$\times\left[c_0^{'}\left(u^*,t,m\right)+X_t^T c^{'}\left(u^*,t,m\right)\right]C_r,$$

where $u^* = d^{rf}(t+m,T-m)$. Note that u^* is only a function of the reference date for the singleton futures, $t+h$, and the length of time between the reference date and the settlement date, which will not change as we vary t or m in the coefficient recursions. That is, for a given set of risk-free zero coupon coefficients, we can apply the same strategy as for simple interest rates to compute futures coefficients. The differentiated coefficients, $c_0^{'}(\cdot)$ and $c^{'}(\cdot)$, can be computed by taking derivatives with respect to u on both sides of Equation (19),

$$c_0^{'}(u,t,h) = c_0^{'}(u,t+1,h-1)+A^{'Q}\left(I_{t+1},c(u,t+1,h-1)+\delta\right)c^{'}\left(u,t+1,h-1\right)$$
$$c^{'}(u,t,h) = B^{'Q}\left(I_{t+1},c(u,t+1,h-1)\right)c^{'}(u,t+1,h-1)$$

for any u, t and any $h>0$. Initial conditions for these differentiated recursions can be found by differentiation of the corresponding initial conditions or by noting that we must have $f(t,0,T)=D(t,T)r_t$. This yields $c_0^{'}\left(u,t,0\right)=0$ and $c^{'}\left(u,t,0\right)$ is the identity matrix. Finally, the derivatives of Laplace coefficients, $A^{Q'}(\cdot)$ and $B^{Q'}(\cdot)$ can be computed directly.

F *Predictability Coefficients*

F.1 Multi-Horizon Laplace Transform

The distribution of future state variables can be characterized explicitly from the multi-horizon conditional Laplace transform,

$$T_X(u,t,h) \equiv E_t\left[\exp(u^T X_{t+h})\right],$$

for any $u\in\mathbb{R}^K$ and $h\geq 1$. The solution is exponential affine,

$$T_X(u,t,h) \equiv E_t\left[T_X(u,t+1,h-1)\right] = \exp\left(A(I_{t+1},z,h)+B(I_{t+1},z,h)^T X_t\right),$$

with coefficients given by

$$A(I_t,z,h+1) = A(I_{t+1},z,h)+A(I_t,B(I_{t+1},z,h))$$
$$B(I_t,z,h+1) = B(I_t,B(I_{t+1},z,h)),$$

for any $h\geq 1$ with initial conditions $A(I_{t+1},z,1)=A(I_{t+1},z)$ and $B(I_{t+1},z,1)=B(I_{t+1},z)$.

F.2 Conditional Expectations

The conditional expectation of linear combinations of the state variables, $C^T X_t$, can be derived at any horizon from the following partial derivative with respect to u,

$$
\begin{aligned}
E_t\left[C^T X_{t+h}\right] &= \left[\frac{\partial}{\partial u} T_X(uC,t,h)\right]_{u=0} \\
&= \left[T_X(uC,t,h)\left(A_2^{'}(I_{t+1},uC,h) + X_t^T B_2^{'}(I_{t+1},uC,h))\right) C\right]_{u=0} \\
&= \left(A_2^{'}(I_{t+1},0,h) + X_t^T B_2^{'}(I_{t+1},0,h))\right) C,
\end{aligned}
$$

where, as before, the derivatives of the multi-horizon coefficients can be obtained by differentiating their respective recursions. The derivatives of the Laplace Transform coefficients under the historical measure can also be computed explicitly.

and the multi-horizon derivatives with respect to the second argument are

$$
\begin{aligned}
A_2^{'}(I_{t+1},0,h) =& A_2^{'}(I_{t+2},0,h-1) + A_2^{'}(I_{t+1},B(I_{t+2},0,h-1))B_2^{'}(I_{t+2},0,h-1)) \\
B_2^{'}(I_{t+1},0,h)) =& B_2^{'}(I_{t+2},B(I_{t+2},0,h-1))B_2^{'}(I_{t+2},0,h-1)),
\end{aligned}
$$

with initial conditions $A_2^{'}(I_{t+1},0,1) = A_2^{'}(I_{t+1},0)$ and $B_2^{'}(I_{t+1},0,1) = B_2^{'}(I_{t+1},0)$.

F.3 Forecasting Fed Funds Rates

The forecasts of target and effective overnight fed funds rates can by computed by setting $C = C_r = [1\ 0\ 0\ 0]^T$ and $C = C_{r+s} = [1\ 1\ 0\ 0]^T$, respectively. We have,

$$
\begin{aligned}
E[r_{t+h}|X_t = x] =& a_r(I_{t+1},h) + b_r(I_{t+1},h)^T X_t \\
E[r_{t+h}|X_t = x] =& a_{r+s}(I_{t+1},h) + b_{r+s}(I_{t+1},h)^T X_t.
\end{aligned}
$$

Figure 2: Forward LIBOR and Futures Rates

LIBOR forward rates and futures rates at maturities of 2, 4 and 6 months. Daily data from January 1994 to July 2007.

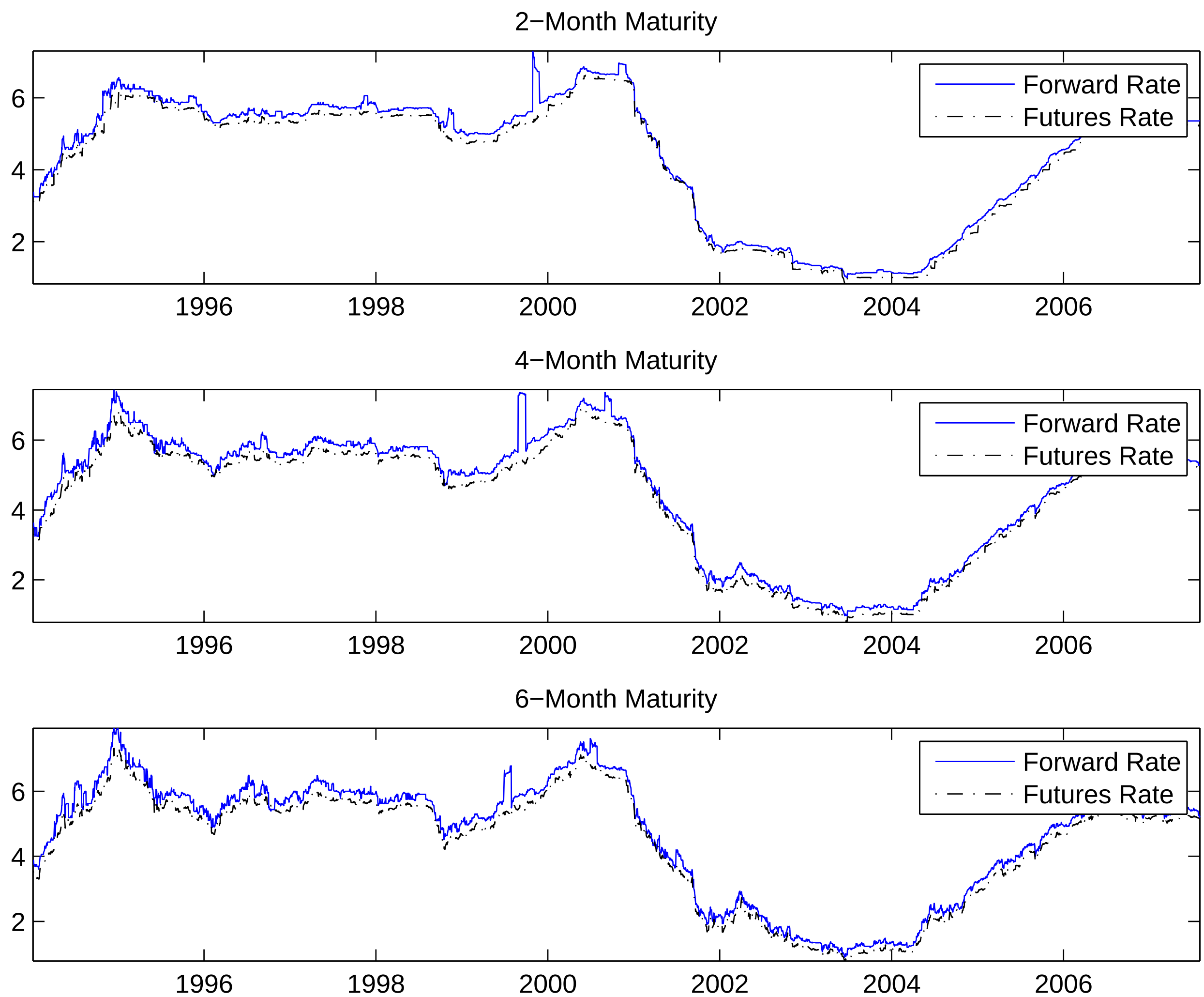


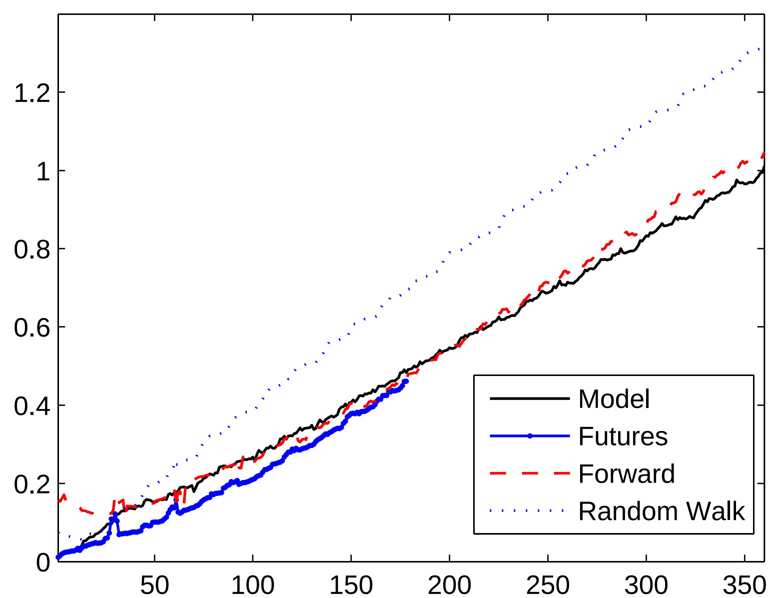
Figure 3: Forecasting Target Rates : RMSE and R^2s

RMSE and R^2 from predictive regressions of monthly target rate average, $\bar{r}_n$ at daily horizons up to 12 months. The predictive regressions are given by

$$
\begin{aligned}
E_t[\bar{r}_n] &= \bar{a}(t,n) + \bar{b}(t,n)^T X_t \\
E_t^{fut}[\bar{r}_n] &= \alpha_h^{fut} + \beta_h^{fut} F(t,n) \\
E_t^{lib}[\bar{r}_n] &= \alpha_h^{lib} + \beta_h^{lib} F_{lib}(t,n),
\end{aligned}
$$

where $\bar{a}(t,n)$ and $\bar{b}(t,n)$ are model-implied coefficients for a time-t forecast of the average effective rate in calendar month n, $\bar{r}_n$, so that h is the number of days between t and the end of month n. $F(t,n)$ and $F_{lib}(t,n)$ are the observed futures and forward LIBOR rates corresponding to calendar month n at time-t. Panel (a) compares the RMSE in percentage (annualized) and the x-axis is the horizon, h, from 1 to 360 days ahead. Panel (b) compares R^2.

(a) RMSE



(b) R^2

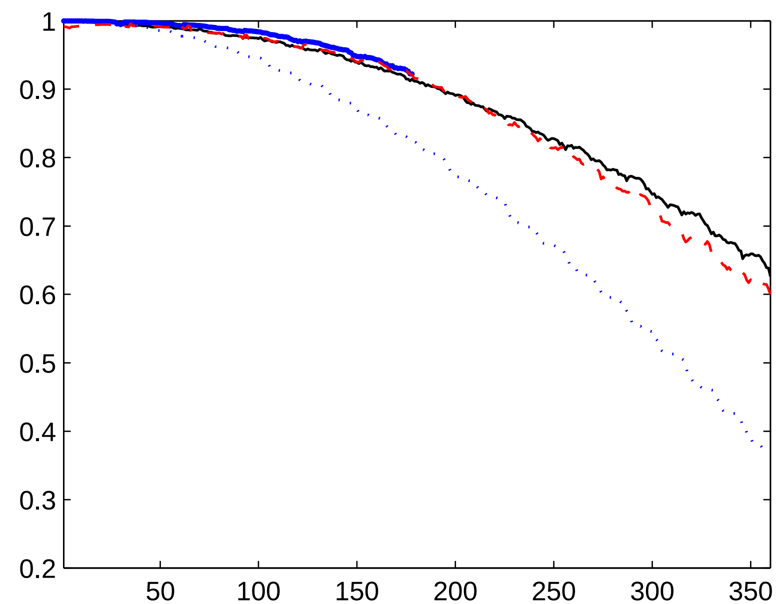
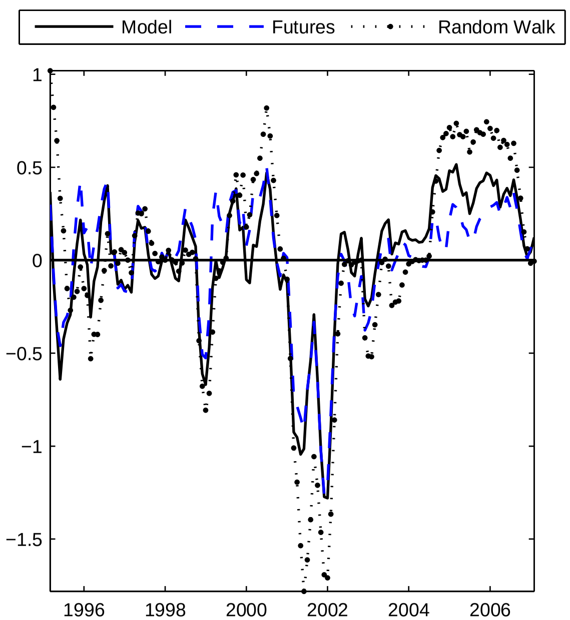


Figure 4: Forecasting Target Rates : Forecast Errors

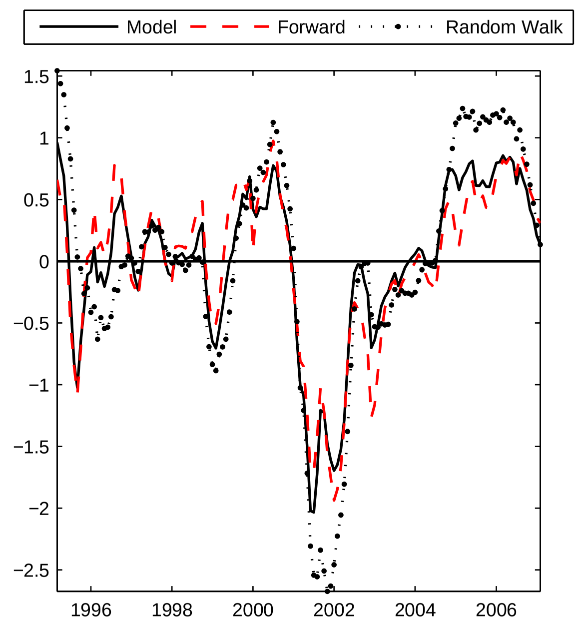
Forecasts errors from predictive regressions of monthly target rate average, $\bar{r}_n$ at daily horizons up to 12 months. The predictive regressions are given by

$$E_t[\bar{r}_n] = \bar{a}(t,n) + \bar{b}(t,n)^T X_t$$
$$E_t^{fut}[\bar{r}_n] = \alpha_h^{fut} + \beta_h^{fut} F(t,n)$$
$$E_t^{lib}[\bar{r}_n] = \alpha_h^{lib} + \beta_h^{lib} F_{lib}(t,n),$$

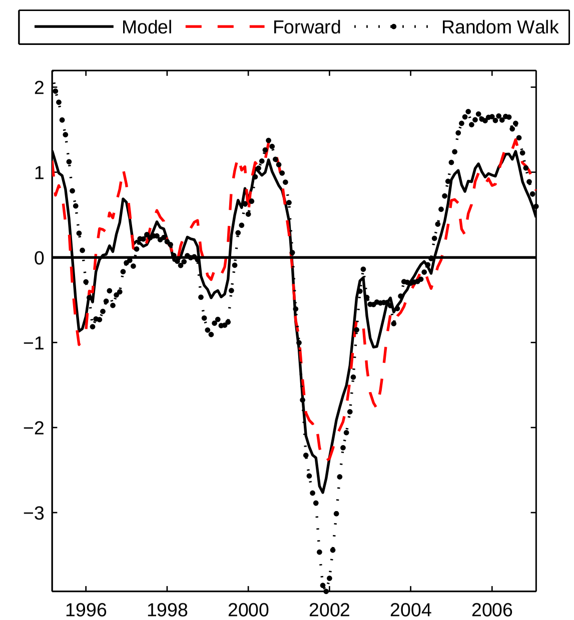
where $\bar{a}(t,n)$ and $\bar{b}(t,n)$ are model-implied coefficients for a time-t forecast of the average effective rate in calendar month n, $\bar{r}_n$, so that h is the number of days between t and the end of month n. $F(t,n)$ and $F_{lib}(t,n)$ are the observed futures and forward LIBOR rates corresponding to calendare month n at time-t. For each observation day, Panel (a) shows the average forecast errors at horizons between 90 and 180 days, Panel (b) at horizons between 180 and 270 days and Panel (c) at horizons between 180 and 270 days.



(a) Horizons from 90 to 180 days



(b) Horizons from 180 to 270 days

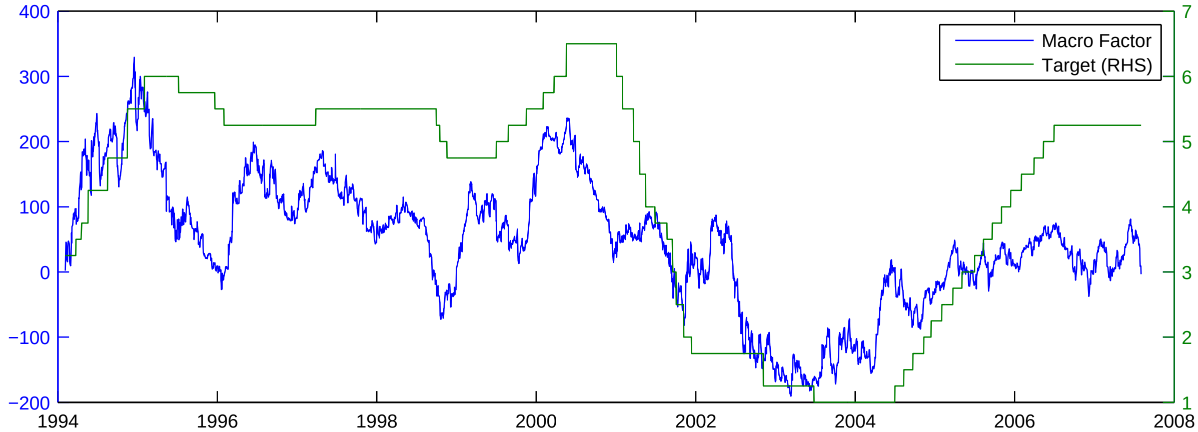


(c) Horizons from 270 days to 360 days

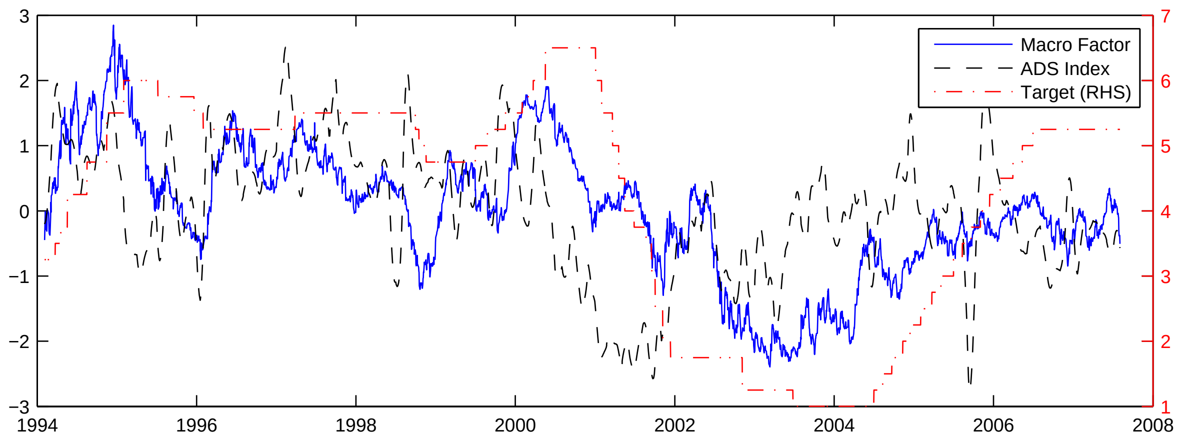
Figure 5: Target Rate, the Macro Factor and Economic Conditions

Target rate, filtered Macro factor and the ADS index of real activity. Panel (a) displays the macro factor and the target rate. Panel (b) displays the ADS index and the target rate. Factors are from QML estimation of the model. ADS index from the Philadelphia Federal Reserve Bank's website.

(a) Target Rate and Macro Factor



(b) ADS Index And Macro Factor



(c) Liquidity And Macro Factor

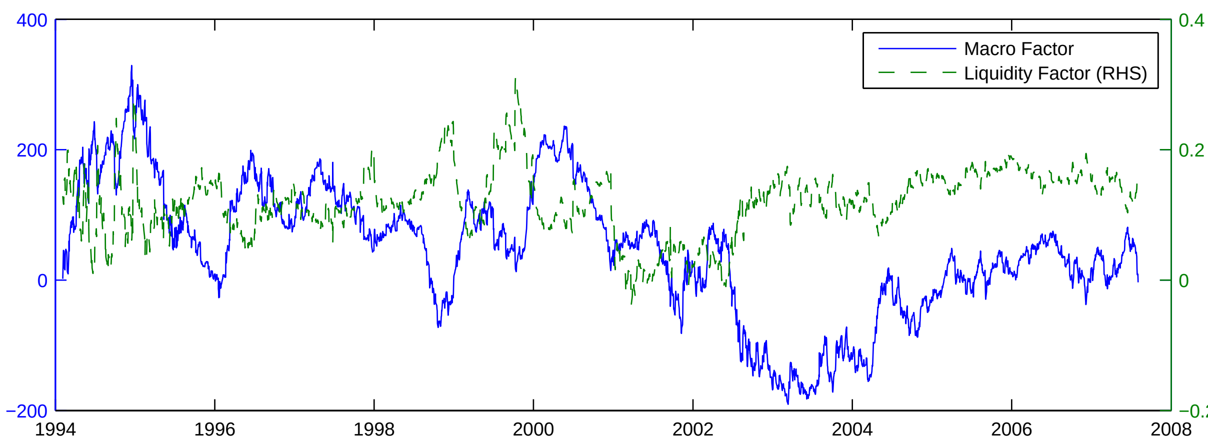
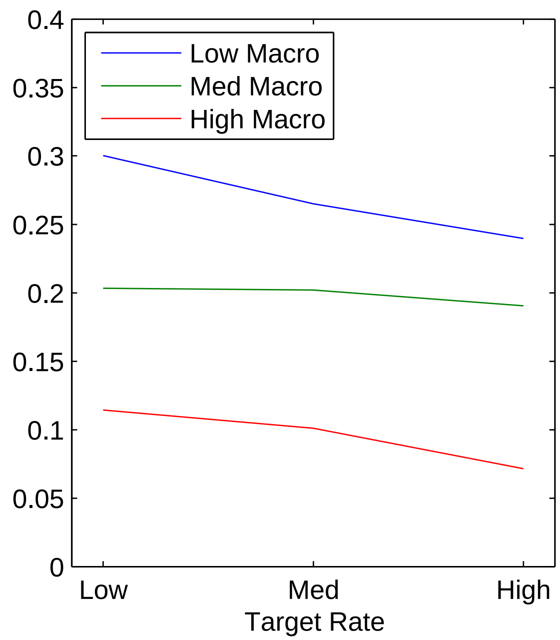


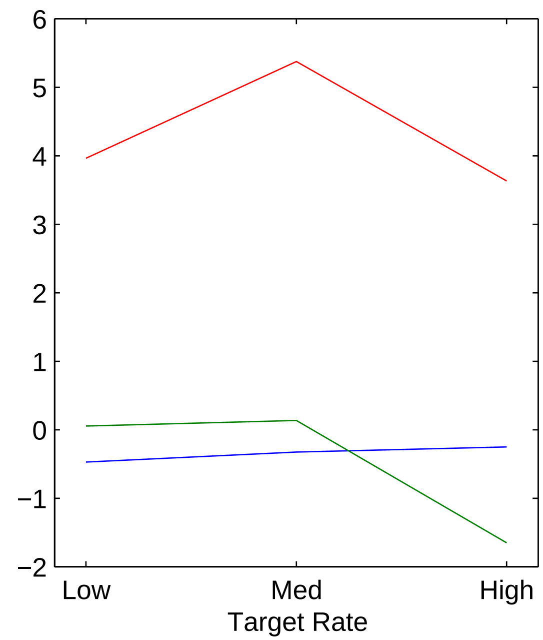
Figure 6: Conditional Moments of Target Rate Changes and State Variables

Variations of the target rate conditional distribution across values of state variables. The sample is divided in three sub-sample along the terciles of the macro factor. Each sub-sample is then divided along its own target rate terciles. There is a total of 9 sub-samples. Panel 6(a) displays variations of conditional variance, Panel 6(b) displays conditional skewness and Panel 6(c) displays conditional kurtosis. For each Panel, a line keeps the macro factor fixed and varies the target rates. Moments are computed using Equation 9-10 and reflect the distribution of the FOMC decision measured on a day preceding a scheduled FOMC meeting. Conditional volatility is measured in annualized percentage. Skewness and Kurtosis are unitless.

(a) Conditional Volatility



(b) Conditional Skewness



(c) Conditional Kurtosis

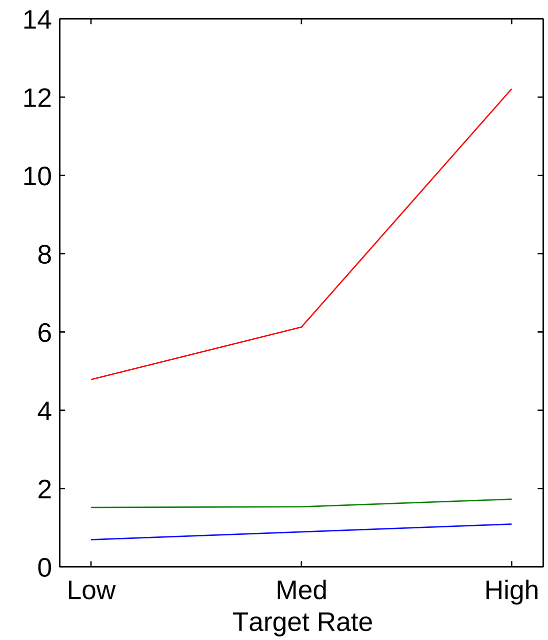
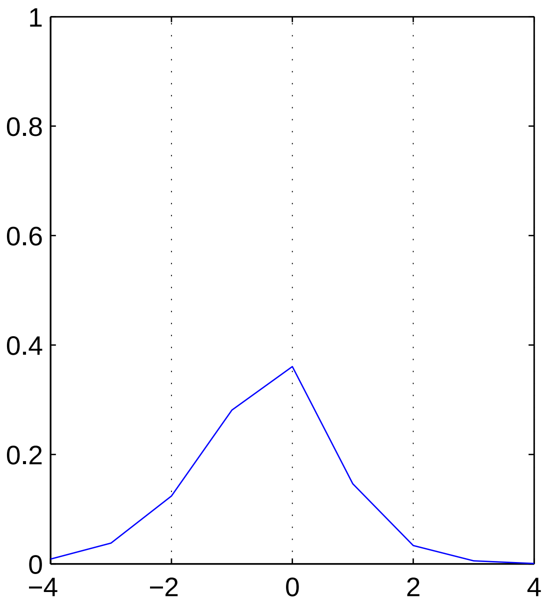


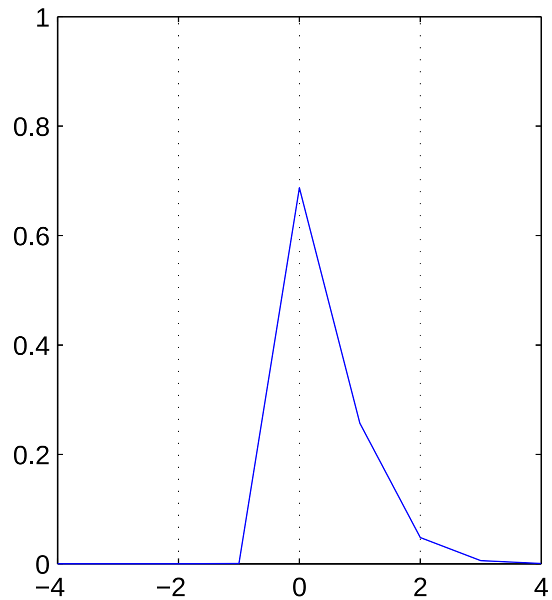
Figure 7: Conditional Distribution of Target Rate Changes

The target rate conditional distributions across values of state variables. The sample is divided along the terciles of the macro factor. Each sub-sample is then divided along its own target rate terciles. There is a total of 9 sub-samples. Panel 7(a) displays the conditional distribution of target changes for the average values of state variables in the (*low*, *low*) sub-sample, Panel 7(b) in the (*med*, *med*) sub-sample and Panel 7(c) in the (*high*, *high*) sub-sample. Conditional probabilities reflect the distribution measured on a day preceding a scheduled FOMC meeting.

(a) Low Macro Factor and Target Rate



(b) Median Macro Factor and Target Rate



(c) High Macro Factor and Target Rate

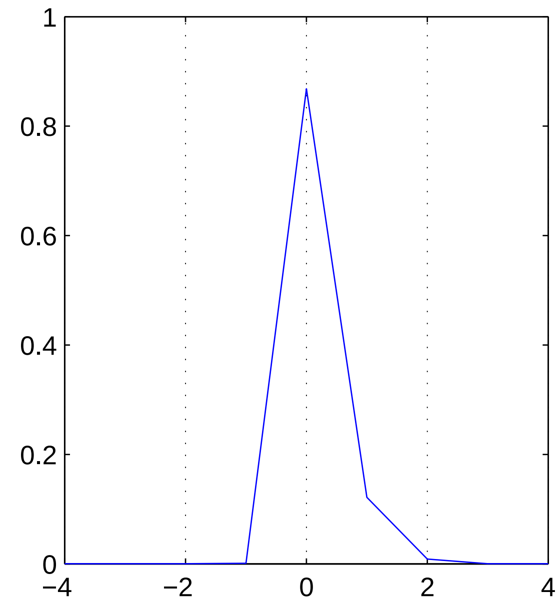


Figure 8: Risk Premium

Panel (a) compares the risk premium predicted by the model for the 6-month ahead contract with the subsequent realized returns on that contract.

(a) Risk Premium And Excess Returns